Real Numbers

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Comprehensive study notes for

Real Numbers

by

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(Math King of Katargam). Master every concept with clear explanations, solved examples, and practice problems.

Key Concepts

Euclid's Division Lemma

For any positive integers a and b, there exist unique whole numbers q and r such that

a = bq + r

where 0 <= r < b. This is the foundation of the Euclidean algorithm.

Euclid's Division Algorithm

To find HCF of two numbers: (1) Apply division lemma to get a = bq + r. (2) If r = 0, HCF = b. (3) Otherwise, apply to (b, r). (4) Repeat until remainder is 0. The last divisor is the HCF.

Fundamental Theorem of Arithmetic

Every composite number can be expressed as a

unique product of primes

(up to order). For example, 32760 = 23 x 32 x 5 x 7 x 13.

Irrational Numbers

A number that cannot be expressed as p/q where p, q are integers and q != 0. Examples: ?2, ?3, ?. The proof that ?2 is irrational uses the

contradiction method

.

Rational Numbers and Decimal Expansions

If the denominator has only prime factors 2 and/or 5, the decimal expansion

terminates

. Otherwise, it

repeats

(recurring). The length of the repeating block divides (q-1).

LCM and HCF using Prime Factorization

For two numbers:

HCF x LCM = a x b

. HCF = product of common prime factors with smallest powers. LCM = product of all prime factors with largest powers.

Important Formulas

Division Lemma

a = bq + r, 0 <= r < b

HCF x LCM

HCF(a,b) x LCM(a,b) = a x b

Prime Factorization

Every composite = product of primes (unique)

Irrational Proof

Assume ?p = a/b (reduced), square both sides, contradiction

Solved Examples

Example 1:

Find HCF of 56 and 72 using Euclid's algorithm.

Solution:

72 = 56 x 1 + 16. 56 = 16 x 3 + 8. 16 = 8 x 2 + 0.

HCF = 8

.

Example 2:

Express 729 as product of primes.

Solution:

729 = 3 x 243 = 3 x 3 x 81 = 3&sup6;. So

729 = 3&sup6;

.

Example 3:

Prove that ?3 is irrational.

Solution:

Assume ?3 = p/q (reduced). Then 3q2 = p2. So p is divisible by 3 => p = 3k. Then 3q2 = 9k2 => q2 = 3k2. So q is divisible by 3. Contradiction as p/q was reduced.

Hence ?3 is irrational.

Practice Questions

Find HCF of 135 and 225 using Euclid's algorithm.

Find LCM and HCF of 12, 15, 21 using prime factorization.

Prove that ?5 is irrational.

Without dividing, check if 51/150 has a terminating decimal expansion.

Show that 3?2 is irrational.

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