Polynomials

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Polynomials

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Comprehensive study notes for

Polynomials

by

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(Math King of Katargam). Learn about algebraic expressions, degrees of polynomials, factorization, and algebraic identities.

1. What is a Polynomial?

An algebraic expression of the form P(x) = anxn + an?1xn?1 + ... + a1x + a0, where an != 0 and n is a whole number, is called a polynomial in variable x.

Examples:

3x2 + 2x - 1, 5y3 - 2y + 7, 4z + 1

Non-examples:

?x + 2 (exponent is 1/2), 1/x + 3 (exponent is -1)

2. Degree of a Polynomial

The highest power of the variable in a polynomial is called its degree.

Polynomial

Degree

Name

5

Constant

2x + 1

1

Linear

3x2 - 2x + 1

2

Quadratic

x3 - 3x2 + 2x - 1

3

Cubic

x4 + 2x3 - x2 + 3x - 5

4

Quartic (Biquadratic)

3. Types of Polynomials (by Number of Terms)

Monomial:

One term (e.g., 3x2, 5, -2xy)

Binomial:

Two terms (e.g., 2x + 3, x2 - 5)

Trinomial:

Three terms (e.g., x2 + 2x - 1)

4. Value and Zero of a Polynomial

Value of a Polynomial:

P(a) is the value obtained by substituting x = a in P(x).

Zero/Root of a Polynomial:

A number a such that P(a) = 0 is called a zero of the polynomial.

Example:

For P(x) = x2 - 5x + 6, P(2) = 4 - 10 + 6 = 0, so x = 2 is a zero.

5. Remainder Theorem

If a polynomial P(x) is divided by (x - a), the remainder is P(a).

Proof:

P(x) = (x - a)Q(x) + R, where R is a constant (since divisor is of degree 1). Substituting x = a gives P(a) = R.

Example:

Find the remainder when x3 - 3x2 + 2x - 1 is divided by (x - 2).

P(2) = 8 - 12 + 4 - 1 =

-1

6. Factor Theorem

(x - a) is a factor of P(x) if and only if P(a) = 0.

Example:

Check if (x - 3) is a factor of x3 - 6x2 + 11x - 6.

P(3) = 27 - 54 + 33 - 6 = 0. Yes, (x - 3) is a factor.

7. Algebraic Identities

Memorize these key identities:

(a + b)2 = a2 + 2ab + b2

(a - b)2 = a2 - 2ab + b2

a2 - b2 = (a + b)(a - b)

(x + a)(x + b) = x2 + (a + b)x + ab

(a + b + c)2 = a2 + b2 + c2 + 2ab + 2bc + 2ca

(a + b)3 = a3 + 3a2b + 3ab2 + b3

(a - b)3 = a3 - 3a2b + 3ab2 - b3

a3 + b3 = (a + b)(a2 - ab + b2)

a3 - b3 = (a - b)(a2 + ab + b2)

8. Factorization Techniques

Method 1: Common Factor

Example: 6x2y + 9xy2 = 3xy(2x + 3y)

Method 2: Grouping

Example: x3 + x2 + x + 1 = x2(x + 1) + 1(x + 1) = (x + 1)(x2 + 1)

Method 3: Splitting the Middle Term

Example: Factorise x2 + 7x + 12. Find p, q such that p + q = 7 and pq = 12. p = 3, q = 4.

= x2 + 3x + 4x + 12 = x(x + 3) + 4(x + 3) =

(x + 3)(x + 4)

Solved Examples

Example 1:

Using identities, evaluate 1022.

Solution:

(100 + 2)2 = 1002 + 2(100)(2) + 22 = 10000 + 400 + 4 =

10404

Example 2:

Factorise x3 - 8.

Solution:

Using a3 - b3 = (a - b)(a2 + ab + b2), where a = x, b = 2:

= (x - 2)(x2 + 2x + 4)

Example 3:

Find the value of k if (x - 1) is a factor of 4x3 + 3x2 - 4x + k.

Solution:

P(1) = 4(1)3 + 3(1)2 - 4(1) + k = 4 + 3 - 4 + k = 3 + k

For (x - 1) to be a factor, P(1) = 0, so 3 + k = 0 ?

k = -3

Practice Questions

Find the degree of the polynomial 5x3 - 2x2 + 7x - 1.

Find the zero of the polynomial P(x) = 2x + 5.

Factorise: x2 + 9x + 18

Factorise: x3 - 3x2 - 9x - 5

Evaluate using identities: 104 x 96

If x + 1/x = 5, find x2 + 1/x2.

Expand: (a - b + c)2

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