Number Systems

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Comprehensive study notes for

Number Systems

by

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(Math King of Katargam). Master the classification of numbers, irrational numbers, real numbers, and their representation on the number line.

Introduction

In this chapter, we extend our understanding of numbers from natural numbers to real numbers. We explore different types of numbers, their properties, and how they are represented on the number line.

1. Types of Numbers

Natural Numbers (N)

: 1, 2, 3, 4, ... (Counting numbers)

Whole Numbers (W)

: 0, 1, 2, 3, 4, ... (Natural numbers + 0)

Integers (Z)

: ..., -3, -2, -1, 0, 1, 2, 3, ... (Whole numbers + negatives)

Rational Numbers (Q)

: Numbers that can be expressed as p/q where p, q are integers and q != 0

Irrational Numbers

: Numbers that cannot be expressed as p/q (e.g., ?2, ?, e)

Real Numbers (R)

: All rational and irrational numbers together

2. Rational Numbers

A number that can be written in the form p/q, where p and q are integers and q != 0, is called a rational number.

Examples:

1/2, 3/4, -5/7, 0, 2 (which is 2/1), 0.333... (which is 1/3)

Properties of Rational Numbers:

Rational numbers are

closed

under addition, subtraction, multiplication, and division (except by zero)

Rational numbers satisfy

commutative, associative, and distributive laws

Between any two rational numbers, there exist infinitely many rational numbers

3. Irrational Numbers

Numbers that cannot be expressed in the form p/q are called irrational numbers.

Examples:

?2 = 1.41421356..., ?3 = 1.73205080..., ? = 3.14159265..., e = 2.71828182...

Key Fact:

The decimal expansion of an irrational number is non-terminating and non-repeating.

Proof that ?2 is Irrational

Assume ?2 = p/q (in simplest form, where p and q have no common factors)

Squaring both sides: 2 = p2/q2 ? p2 = 2q2

This means p2 is even, so p must be even

Let p = 2r, then (2r)2 = 2q2 ? 4r2 = 2q2 ? q2 = 2r2

This means q2 is even, so q must be even

But if both p and q are even, they have a common factor of 2 -- contradicting our assumption

Therefore, ?2 cannot be expressed as p/q

4. Real Numbers and Their Decimal Expansions

Type	Decimal Expansion	Example
Terminating	Finite number of digits after decimal	1/4 = 0.25
Non-terminating Repeating	Infinite digits with a repeating pattern	1/3 = 0.333...
Non-terminating Non-repeating	Infinite digits with no pattern	?2 = 1.41421...

5. Representing Real Numbers on the Number Line

Every real number corresponds to a unique point on the number line, and every point on the number line corresponds to a unique real number.

To locate ?2 on the number line:

Draw a number line. Mark 0 at the center and 1 unit to the right

Construct a right-angled triangle with base = 1 unit and height = 1 unit

The hypotenuse = ?(12 + 12) = ?2

Using a compass, transfer the length of the hypotenuse onto the number line

Similarly, we can locate ?3, ?5, and other irrational numbers on the number line using geometric constructions.

6. Operations on Real Numbers

Important Rules:

?a x ?b = ?(ab) (when a, b >= 0)

?a / ?b = ?(a/b) (when a >= 0, b > 0)

(?a + ?b)(?a - ?b) = a - b

(a + ?b)(a - ?b) = a2 - b

(?a + ?b)2 = a + b + 2?(ab)

7. Rationalising the Denominator

To simplify expressions with irrational denominators, we multiply numerator and denominator by the conjugate.

Example:

Simplify 1/(?3 - ?2)

Multiply numerator and denominator by (?3 + ?2):

= 1/(?3 - ?2) x (?3 + ?2)/(?3 + ?2)

= (?3 + ?2)/((?3)2 - (?2)2)

= (?3 + ?2)/(3 - 2) = ?3 + ?2

8. Laws of Exponents for Real Numbers

For any real numbers a, b and rational numbers m, n:

am x an = am+n

am / an = am-n

(am)n = amn

am x bm = (ab)m

a0 = 1

a-m = 1/am

a1/n = n?a

Solved Examples

Example 1:

Classify the following numbers as rational or irrational: ?5, 0.333..., 22/7, ?, 0.1010010001...

Solution:

?5 = 2.23606... ->

Irrational

(non-terminating, non-repeating)

0.333... ->

Rational

(repeating decimal = 1/3)

22/7 ->

Rational

(in the form p/q)

? = 3.14159... ->

Irrational

0.1010010001... ->

Irrational

(no repeating pattern)

Example 2:

Simplify (?5 + ?3)(?5 - ?3)

Solution:

Using the identity (a + b)(a - b) = a2 - b2

= (?5)2 - (?3)2 = 5 - 3 =

2

Example 3:

Rationalise the denominator of 1/(?5 + ?3)

Solution:

Multiply numerator and denominator by (?5 - ?3):

= (?5 - ?3)/[(?5)2 - (?3)2] = (?5 - ?3)/(5 - 3) =

(?5 - ?3)/2

Practice Questions

Find four rational numbers between 3 and 4.

Express 0.123 (0.12333...) in the form p/q.

Simplify: (3 + ?3)(2 + ?2)

Rationalise the denominator: 1/(?7 - ?6)

If ?2 = 1.414, find the value of ?(?2 - 1/?2)

Prove that ?3 is irrational.

Find the value of a and b if (?3 - 1)/(?3 + 1) = a + b?3

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