OPEN STUDENT FOUNDATION
STD 10 : MATHS
IMPORTANT QUESTION DAY 10

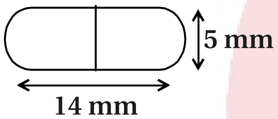
Section A

- **Write the answer of the following questions. [Each carries 4 Marks]** **[40]**

1. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel. (take $\pi = \frac{22}{7}$)

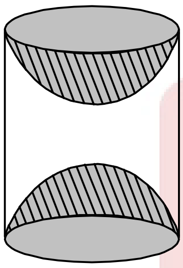
2. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy. (take $\pi = \frac{22}{7}$)

3. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see Fig.). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area. (take $\pi = \frac{22}{7}$)



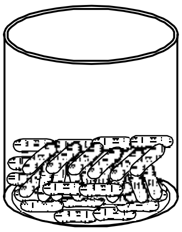
4. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of ₹ 500 per m^2. (Note that the base of the tent will not be covered with canvas.)(take $\pi = \frac{22}{7}$)

5. A wooden article was made byscooping out a hemisphere from each end of a solid cylinder, as shown in Fig. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article. (take $\pi = \frac{22}{7}$)



6. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.) (take $\pi = \frac{22}{7}$)

7. A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (see Figure). (take $\pi = \frac{22}{7}$)



8. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel. (take $\pi = \frac{22}{7}$)

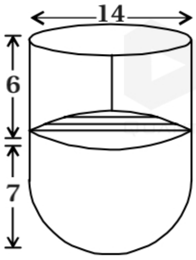
9. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm. (take $\pi = \frac{22}{7}$)

10. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements, and $\pi = 3.14$.

Section A

- **Write the answer of the following questions. [Each carries 4 Marks]** **[40]**

1. A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel. (take $\pi = \frac{22}{7}$)

➤ 

For hollow cylinder :

Diameter $d = 14$ cm $\Rightarrow$ radius $r = 7$ cm

Height $h = (13 - 7) = 6$ cm

The curved surface area of cylinder $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 7 \times 6 \text{ cm}^2$$

$$= 264 \text{ cm}^2$$

For hemisphere :

Diameter $d = 14$ cm $\Rightarrow$ Radius $r = \frac{d}{2} = \frac{14}{2} = 7$ cm

The curved surface area of hemisphere $= 2\pi r^2$

$$= 2 \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2$$

$$= 308 \text{ cm}^2$$

$\therefore$ The inner surface area of the vessel

= The curved surface area of cylinder + the curved surface area of hemisphere

$= (264 + 308)$ cm^2

$= 572$ cm^2

2. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy. (take $\pi = \frac{22}{7}$)

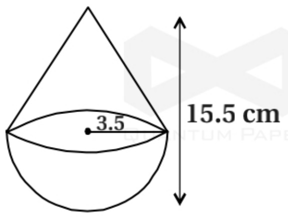
➤ Radius of the cone = Radius of the

hemisphere $= r = 3.5$ cm

The height of the cone = The total height of the toy – the height of the hemisphere

$$= (15.5 - 3.5) \text{ cm}$$

$= 12$ cm



Slant height of the cone $l = \sqrt{h^2 + r^2}$

$= \sqrt{(12)^2 + (3.5)^2}$ cm

$= \sqrt{144 + 12.25}$ cm

$= \sqrt{156.25}$ cm

$\therefore\ l = 12.5$ cm

The total surface area of the toy

= the curved surface area of the cone + the curved surface area of the hemisphere

$= \pi rl + 2\pi r^2$

$= \pi r\ (l + 2r)$

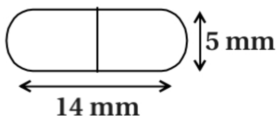
$= \frac{22}{7} \times \frac{35}{10} (12.5 + 2 \times 3.5)$ cm^2

$= \frac{22}{7} \times \frac{35}{10} \times 19.5$ cm^2

$= 11 \times 19.5$ cm^2

$\therefore$ Hence, the total surface area of the toy is 214.5 cm^2

3. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see Fig.). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area. (take $\pi = \frac{22}{7}$)



➤ The length of the medicine capsule = 14 mm

The diameter of the capsule = 5 mm

$\therefore$ radius $r = \frac{\text{Diameter}}{2} = \frac{5}{2} = 2.5$ mm

The area of the two hemisphere at the end of the capsule $= 2 \times 2\pi r^2$

$= 2 \times \left[2 \times \frac{22}{7} \times \frac{25}{10} \times \frac{25}{10}\right]$ mm^2

$= 2 \times \left[2 \times \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2}\right]$ mm^2

$= 2 \times \left[\frac{11 \times 5 \times 5}{7}\right]$ mm^2

$= 2 \times \left[\frac{11 \times 25}{7}\right]$ mm^2

$= 2\left[\frac{275}{7}\right]$ mm^2

The curved surface area of the cylinder $A = 2\pi rh$

$A = 2 \times \frac{22}{7} \times \frac{5}{10} \times 9 \quad (\because 14 - 2.5 + 2.5 = 9)$

$= \frac{110 \times 9}{7}$ mm^2

The surface area of the capsule = The area of the two hemisphere + the curved surface area of cylinder

$= \left(\frac{2 \times 275}{7}\right)$ mm^2 + $\left(\frac{110 \times 9}{7}\right)$ mm^2

$= \left(\frac{550 + 990}{7}\right)$ mm^2

$= \left(\frac{1540}{7}\right)$ mm^2

= 220 mm^2

Hence, the surface area of the capsule is 220 mm^2

➤ **Second Method :**

The surface area of capsule = πdH

Where d is diameter of capsule

H = Length of capsule

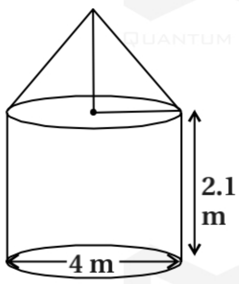
$= \frac{22}{7} \times 5 \times 14$

= 220 મિમી^{2}

4. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of ₹ 500 per m^2. (Note that the base of the tent will not be covered with canvas.)(take $\pi = \frac{22}{7}$)

➤ The height of the cylindrical part h = 2.1 m

The diameter of the cylindrical part = 4 m



$\therefore$ Radius $= \frac{\text{Diameter}}{2} = \frac{4}{2} = 2$ m

The radius of the conical part r = 2m

The slant height of the conical part l = 2.8 m

The total curved surface area of the tent =

The curved surface area of the cylinder + the curved surface area of the cone.

$= 2\pi rh + \pi rl$

$= \pi r\ (2h + l)$

$= \frac{22}{7} \times 2\ (2 \times 2.1 + 2.8)\ m^2$

$= \frac{22 \times 2}{7}\ (4.2 + 2.8)\ m^2$

$= \frac{22 \times 2}{7}\ (7)\ m^2$

$= \frac{44}{7} \times 7\ m^2$

$= 44\ m^2$

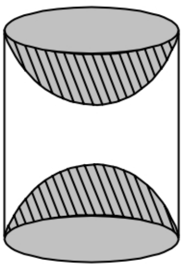
$1\ m^2$ = ₹ 500

$\therefore\ 44\ m^2$ = (?)

$\therefore\ 500 \times 44$ = ₹ 22000

Hence, the cost of the canvas used to make tent is ₹ 22000 and the area of the canvas is 44 m^2.

5. A wooden article was made byscooping out a hemisphere from each end of a solid cylinder, as shown in Fig. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article. (take $\pi = \frac{22}{7}$)



➤ Radius of the cylinder $r = 3.5$ cm

Height of the cylinder $h = 10$ cm

The total surface area of the cylinder $= 2\pi rh + 2\pi r^2$

$= 2\pi r\ (h + r)$

$= 2 \times \frac{22}{7} \times \frac{35}{10}\left(10 + \frac{35}{10}\right)\text{cm}^2$

$= 22\left(\frac{135}{10}\right)\text{cm}^2$

$= 297\ \text{cm}^2$

The curved surface area of two hemisphere $= (2\pi r^2) \times 2$

$= \left(2 \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10}\right)\ \text{cm}^2 \times 2$

$= 154\ \text{cm}^2$

The area of the base of hemisphere is πr^2

$\therefore$ The area of the base of two hemisphere $= 2 \times 2\pi r^2$

$= 2 \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10}\ \text{cm}^2$

$= 77 \text{ cm}^2$

The total surface area of the article $= (297 + 154 - 77) \text{ cm}^2$

$= (451 - 77) \text{ cm}^2$

$= 374 \text{ cm}^2$

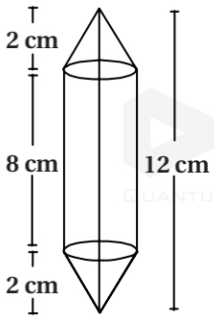
6. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.) (take $\pi = \frac{22}{7}$)

➤ For both the cones,

Radius $r = \frac{3}{2}$ cm

Height $h = 2$ cm

For cylinder Radius $r = \frac{3}{2}$ cm



Height H = Height of the model – 2 × height of the cone

$= (12 - 2 \times 2)$ cm

$= 8$ cm

The volume of the air contained in the model = The volume of the model (cylinder + cone + cone)

$$V = \pi r^2 H + 2 \times \frac{1}{3} \pi r^2 h$$

$$= \pi r^2 \left(H + \frac{2}{3} h\right]$$

$$= \frac{22}{7} \times \left(\frac{3}{2}\right)^2 \left[8 + \frac{2}{3} \times 2\right] \text{ cm}^3$$

$$= \frac{22}{7} \times \frac{9}{4} \left[\frac{8}{1} + \frac{4}{3}\right] \text{ cm}^3$$

$$= \frac{22}{7} \times \frac{9}{4} \left[\frac{24 + 4}{3}\right] \text{ cm}^3$$

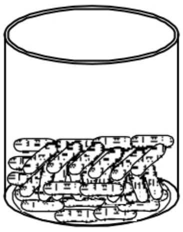
$$= \frac{22}{7} \times \frac{9}{4} \times \frac{28}{3} \text{ cm}^3$$

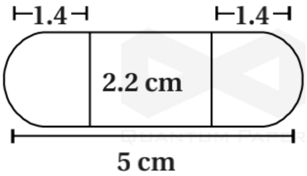
$\therefore$ $V = 66 \text{ cm}^3$

Hence, the volume of air contained in the model is 66 cm^3.

7. A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with

length 5 cm and diameter 2.8 cm (see Figure). (take $\pi = \frac{22}{7}$)



➤ 

The radius of the cylindrical part of gulab jamun = Radius of hemisphere at both the ends.

Diameter = 2.8 cm

$\therefore$ radius $= \frac{2.8}{2} = 1.4$ cm

The height of the cylindrical part of gulab jamun

H = Total height − 2 × radius of hemisphere

$= 5 - 2 \times 1.4$

$= 5 - 2.8 = 2.2$ cm

The volume of a gulab jamun = the volume of cylinder + 2 × the volume of hemisphere

$$= \pi r^2 H + 2 \times \frac{2}{3}\pi r^3$$

$$= \pi r^2 \left(H + \frac{4}{3} r\right)$$

$$= \frac{22}{7} \times 1.4 \times 1.4 \left[2.2 + \frac{4}{3} \times 1.4\right] \text{ cm}^3$$

$$= 6.16\ [4.07] \text{ cm}^3$$

$$= 25.0712 \text{ cm}^3$$

Hence, the volume of a gulab jamun = 25.0712

$\therefore$ the volume of 45 gulab jamun = 45 × 25.0712

$= 1128.204$ cm^3

A gulab jamun, contains sugar syrup up to about 30 % of its volume.

$\therefore$ The volume of sugar syrup contains in 45 gulab jamuns = 30% of the volume of 45 gulab jamun.

$$= \frac{30}{100} \times 1128.204$$

$$= 338.4612 \text{ cm}^3$$

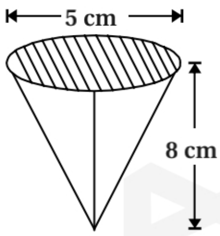
$$= 338 \text{ cm}^3 \text{ (Approximate)}$$

8. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel. (take $\pi = \frac{22}{7}$)

➤ A vessel is in the form of an inverted cone

Its radius R = 5 cm

and height H = 8 cm



Volume of the conical vessel $= \frac{1}{3}\pi R^2 H$

$= \frac{1}{3} \times \pi (5)^2 (8)$

$= \frac{200}{3} \pi \text{ cm}^3$

Volume of a lead shot $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi (0.5)^3$

$= \frac{0.5\pi}{3} \text{ cm}^3$

When the lead shots are dropped, one-fourth of the water flows out

$\therefore$ Volume of the water $= \frac{1}{4} \times \frac{200\pi}{3} = \frac{50}{3}\pi \text{ cm}^3$

The number of lead shots × the volume of a lead shot = volume of the water

$n \times \frac{0.5\pi}{3} = \frac{50}{3}\pi$

$\therefore n \times \frac{\pi}{6} = \frac{50}{3}\pi$

$\therefore \frac{n}{6} = \frac{50}{3}$

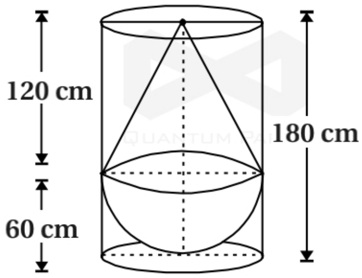
$\therefore 3n = 300$

$\therefore n = \frac{300}{3}$

$\therefore n = 100$

Hence, the number of lead shots dropped in the vessel is 100.

9. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm. (take $\pi = \frac{22}{7}$)

➤ 

The volume of water left in the cylinder

= Volume of cylinder – (Volume of cone + Volume of hemisphere)

$= \pi r^2 H - \left[\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3\right]$

$= \pi r^2 \left[H - \frac{1}{3}h - \frac{2}{3}r\right]$

$= \frac{22}{7} \times (60)^2 \times \left[180 - \frac{1}{3} \times 120 - \frac{2}{3} \times 60\right] \text{ cm}^3$

$= \frac{22}{7} \times 60 \times 60 \times (180 - 40 - 40) \text{ cm}^3$

$= \frac{22}{7} \times 60 \times 60 \times 100 \text{ cm}^3$

$= \frac{22}{7} \times \frac{60 \times 60 \times 100}{(100)^3} \text{ m}^3$

$= \frac{22}{7} \times \frac{60 \times 60 \times 100}{100 \times 100 \times 100} \text{ m}^3$

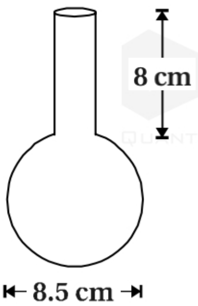
$= \frac{22 \times 6 \times 6}{7 \times 100} \text{ m}^3$

The volume of water left in the cylinder $= \frac{22 \times 36}{7 \times 100} = \frac{792}{7 \times 100} = \frac{792}{700}$

$= 1.131 \text{ m}^3$ (Approximate)

Hence, the volume of water left in the cylinder is 1.131 m^3 (Approximate).

10. A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements, and $\pi = 3.14$.

➤ 

➤ The volume of cylindrical neck

$V = \pi r^2 h$

$= 3.14 \times 1^2 \times 8 \text{ cm}^3$

$= \frac{314}{100} \times 8 \text{ cm}^3$

Radius of the sphere $R = \frac{8.5}{2}$ cm

Volume of the sphere $V = \frac{4}{3}\pi R^3$

$$= \frac{4}{3} \times \frac{314}{100} \times \frac{85}{20} \times \frac{85}{20} \times \frac{85}{20} \text{ cm}^3$$

The volume of the spherical glass vessel

= The volume of the cylindrical neck + the volume of the spherical part.

$$= \left[\frac{314}{100} \times 8\right] \text{cm}^3 + \left[\frac{314}{100} \times \frac{4}{3} \times \frac{85 \times 85 \times 85}{8000}\right] \text{cm}^3$$

$$= \frac{314}{100}\left[8 + \frac{4 \times 85 \times 85 \times 85}{24000}\right] \text{ cm}^3$$

$$= \frac{314}{100}\left[8 + \frac{614125}{6000}\right] \text{ cm}^3$$

$$= \frac{314}{100}\left[\frac{48000 + 614125}{6000}\right] \text{ cm}^3$$

$$= \frac{314}{100}\left[\frac{662125}{6000}\right] \text{ cm}^3$$

$$= \frac{314}{100} \times \frac{5297}{48} \text{ cm}^3$$

$$= \frac{157}{100} \times \frac{5297}{24} \text{ cm}^3$$

$$= \frac{831629}{2400} \text{ cm}^3$$

$= 346.51 \text{ cm}^3$ (Approx)

Hence, she is not correct because $345 \text{ cm}^3 \neq 346.51 \text{ cm}^3$.