OPEN STUDENT FOUNDATION
STD 10 : MATHS
IMPORTANT QUESTION DAY 4

CHAPTER : 5

Date : 21/02/24

Section A

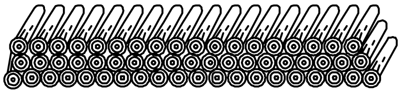
- **Write the answer of the following questions. [Each carries 2 Marks]** **[18]**

1. Find the sum of first 22 terms of an AP in which $d = 7$ and 22nd term is 149.
2. Find the sum of the given APs : 2, 7, 12, ..., to 10 terms.
3. Find the sum of the given APs : 0.6, 1.7, 2.8, ..., to 100 terms
4. Find the sum : $(-5) + (-8) + (-11) + \dots + (-230)$
5. In an AP : given $a = 5$, $d = 3$, $a_n = 50$ find n and S_n
6. In an AP : Given $a_{12} = 37$, $d = 3$ find a and S_{12}.
7. Find the sum of the first 15 multiples of 8.
8. Find the sum of the first 40 positive integers divisible by 6.
9. Which term of the AP : 3, 8, 13, 18, ..., is 78 ?

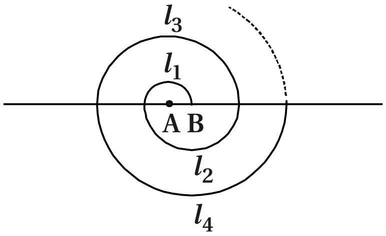
Section B

- **Write the answer of the following questions. [Each carries 3 Marks]** **[51]**

10. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.
11. Which term of the AP : 3, 15, 27, 39, ... will be 132 more than its 54th term ?
12. How many three-digit numbers are divisible by 7 ?
13. The sum of the 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.
14. How many terms of the A.P : 9, 17, 25, ... must be taken to give a sum of 636 ?
15. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.
16. A sum of ₹ 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹ 20 less than its preceding prize, find the value of each of the prizes.
17. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students ?
18. 200 logs are stacked in the following manner : 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see Fig.). In how many rows are the 200 logs placed and how may logs are in the top now ?



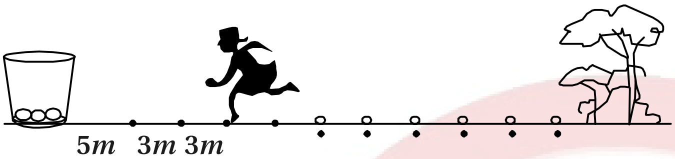
19. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 2.0 cm, ... as shown in Fig. 5.4. What is the total length of such a spiral made up of thirteen consecutive semicircles ? $\left(\text{Take } \pi = \frac{22}{7}\right)$



[Hints : Length of successive semicircles is l_1, l_2, l_3, l_4, ··· with centres at A, B, A, B respectively.

20. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

21. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (see Fig.).



A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

[Hint : To pick up the first potato and the second potato, the total distance (in metres) run by a competitor is $2 \times 5 + 2 \times (5 + 3)$]

22. Ramkali saved ₹ 5 in the first week of a year and then increased her weekly savings by ₹ 1.75. If in the nth week, her weekly savings become ₹ 20.75, find n.

23. Find the 20th term from the last term of the AP : 3, 8, 13, ..., 253.

24. Two APs have the same common difference. The difference between their 100^{th} terms is 100, what is the difference between their 1000th terms ?

25. In an AP : Given $a = 8$, $a_n = 62$ and $S_n = 210$ find n and d.

26. Check whether –150 is a term of the AP : 11, 8, 5, 2 ... First term $a = 11$

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Section A

● **Write the answer of the following questions.** [Each carries 2 Marks] **[18]**

1. Find the sum of first 22 terms of an AP in which $d = 7$ and 22nd term is 149.

➤ Here $d = 7$, $a_{22} = 149$, $n = 22$ first term $= a$

$a_n = a + (n - 1)d$

$\therefore \; a_{22} = a + (22 - 1)d$

$\therefore \; 149 = a + 21(7)$

$\therefore \; 149 = a + 147$

$\therefore \; a = 2$

Sum of first n terms

$S_n = \frac{n}{2}(a + l)$

$\therefore \; S_{22} = \frac{22}{2}(2 + 149)$

$= 11(151)$

$= 1661$

So, $a = 2$ and sum of first 22 terms is 1661.

2. Find the sum of the given APs : 2, 7, 12, ..., to 10 terms.

➤ In the given AP

First term $a = 2$

Common difference = 7 – 2

$n = 10$

Now, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore \; S_{10} = \frac{10}{2}[2(2) + (10 - 1)5]$

$= 5\,[4 + 45]$

$= 5\,[49]$

$= 245$

So, the sum of first 10 terms of the given AP is 245

3. Find the sum of the given APs : 0.6, 1.7, 2.8, ..., to 100 terms

➤ For given AP $a = 0.6$, $d = 1.7 - 0.6 = 1.1$ and $n = 100$

➤ $S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore \; S_{100} = \frac{100}{2}[2 \times 0.6 + (100 - 1)\; 1.1]$

$= 50\ [1.2 + (99)\ 1.1]$

$= 50\ [1.2 + 108.9]$

$= 50\ [110.1]$

$= 50 \times 110.1$

$\therefore\ S_{100} = 5505$

So, the sum of the first 100 terms of the given AP is 5505.

4. Find the sum : $(-5) + (-8) + (-11) + \ldots + (-\ 230)$

➤ Here, first term $a = -5$

Common difference $d = (-8) - (-5)$

$= -8 + 5$

$= -3$

Let n^{th} term $a_n = -230$

$\therefore\ a_n = a + (n - 1)d$

$\therefore\ -230 = -5 + (n - 1)\ (-3)$

$\therefore\ (-230) = -\ 5 - 3n + 3$

$\therefore\ -230 = -3n - 2$

$\therefore\ 3n = 228$

$\therefore\ n = 76$

There are $n = 76$ terms in the given AP

➤ Sum of first n terms of the A. P. $S_n = \frac{n}{2}\ [a + a_n]$

$S_{76} = \frac{76}{2}[-5 + (-230)]$

$= 38\ [-235]$

$= -\ 8930$

So, the sum of first If terms of the given AP is – 8936

5. In an AP : given $a = 5$, $d = 3$, $a_n = 50$ find n and S_n

➤ $a = 5$, $d = 3$, $a_n = 50$

Now, $a_n = a + (n - 1)d$

$\therefore\ 50 = 5 + (n - 1)\ 3$

$\therefore\ 50 = 5 + 3n - 3$

$\therefore\ 50 = 3n + 2$

$\therefore\ 50 - 2 = 3n$

$\therefore\ 48 = 3n$

$\therefore\ n = 16$

➤ $S_n = \frac{n}{2}\ (a + l)$

$\therefore S_{16} = \frac{16}{2} (5 + 50)$

$= 8 (55)$

Thus, $n = 16$ and $S_n = S_{16} = 440$

6. In an AP : Given $a_{12} = 37$, $d = 3$ find a and S_{12}.

➤ Here, $a_{12} = 37 = l$, $n = 12$, $d = 3$, $a = ?$

Now, $a_n = a + (n - 1)d$

$\therefore a_{12} = a + (12 - 1) 3$

$\therefore 37 = a + 33$

$\therefore a = 4$

➤ $S_n = \frac{n}{2} (a + l)$

$S_{12} = \frac{12}{2} (4 + 37)$

$\therefore S_{12} = 6 (41) = 246$

Thus, $a = 4$ and $S_{12} = 246$

7. Find the sum of the first 15 multiples of 8.

➤ Multiples of 8 are 8, 16, 24, 32 ....

which are in AP

Here, $a = 8$, $l = 120$, $n = 15$

➤ $S_n = \frac{n}{2} (a + l)$

$= \frac{15}{2} (8 + 120)$

$= \frac{15}{2} (128)$

$= \frac{15}{2} \times 128$

$= 15 \times 64$

$\therefore S_{15} = 960$

Hence, the sum of the first 15 multiples of 8 is 960

8. Find the sum of the first 40 positive integers divisible by 6.

➤ Positive integers divisible by 6 are 6, 12, 18, 24 ... which are in AP

Here, first term $a = 6$,

and common difference $d = 6$,

we have to find the sum of first 40 terms. So, $n = 40$

Now, sum of the first n terms of an AP,

$S_n = \frac{n}{2} [2a + (n - 1)d]$

$\therefore\ S_{40} = \frac{40}{2}\ [2(6) + (40 - 1)6]$

$= 20\ [12 + (39)6]$

$= 20\ [12 + 234]$

$= 20\ [246]$

$= 4920$

Hence, the sum of the first 40 positive integers divisible by 6 is 4920.

9. Which term of the AP : 3, 8, 13, 18, ..., is 78 ?

➤ Here First term $a = 3$

Common difference $d = 8 - 3 = 5$

Let the n^{th} term of an A. P. be 78.

➤ $a_n = 78$

$\therefore\ a + (n - 1)d = 78$

$\therefore\ 3 + (n - 1)\ (5) = 78$

$\therefore\ 3 + 5n - 5 = 78$

$\therefore\ 5n = 78 + 2$

$\therefore\ 5n = 80$

$\therefore\ n = 16$

Hence, the 16^{th} term of the given AP is 78.

Section B

● **Write the answer of the following questions.** [Each carries 3 Marks] **[51]**

10. Find the 31^{st} term of an AP whose 11^{th} term is 38 and the 16^{th} term is 73.

➤ Let the first term of an A. P. be a and its common difference be d.

Given that 11^{th} term is 38 and 16^{th} term is 73.

$a_{11} = 38$

$\therefore\ a_{11} = 38$

➤ $a_n = a + (n - 1)d$

Taking $n = 11$

$a_{11} = a + (11 - 1)d$

$\therefore\ 38 = a + 10d$ ...(i)

➤ $a_{16} = 73.$

$\therefore\ a_{16} = 73$

➤ $a_n = a + (n - 1)d$

Taking $n = 16$

$a_{16} = a + (16 - 1)d$

$\therefore\ 73 = a + 15d$ ...(ii)

$73 = a + 15d$ (ii)

$38 = a + 10d$ (i)

$-\quad -\quad -$

Subtract : $35 = 5d$

$\therefore\ d = 7$

➤ Substituting $d = 7$ in equation (i) we have $a + 10d = 38$

$a + 10d = 38$

$\therefore\ a + 10(7) = 38$

$\therefore\ a = 38 - 70$

$\therefore\ a = -32$

Now $a_{31} = a + (31 - 1)d$

$= (-32) + 30\ (7)\quad (\because\ a = -32 \text{ and } d = 7)$

$= -32 + 210$

$= 176$

Hence, the 31st term of the given AP is 178.

11. Which term of the AP : 3, 15, 27, 39, ... will be 132 more than its 54th term ?

➤ First term $a = 3$

Common difference $d = 15 - 3 = 12$

➤ $a_n = a + (n - 1)d$

$a_{54} = a + (54 - 1)d \qquad (\because \text{ Taking } n = 54)$

$= 3 + 53\ (12)$

$= 3 + 636 = 639$

Let n^{th} term of the given A. P.

Will be 132 more than its 54 the term

$\therefore\ a_n = a_{54} + 132$

$\therefore\ a_n = 639 + 132 = 771$

➤ $a_n = a + (n - 1)d$

$\therefore\ 771 = 3 + (n - 1)12$

$\therefore\ 771 = 3 + 12n - 12$

$\therefore\ 771 = 12n - 9$

$\therefore\ 771 + 9 = 12n$

$\therefore\ 780 = 12n$

$\therefore\ n = 65$

Hence, 65th term will be 132 more than its 54th term.

12. How many three-digit numbers are divisible by 7 ?

➤ The smallest three digit number is 100 and the largest three digit number is 999

First three digit number divisible by 7, First three digit number divisible by 7 = 100 + 5 = 105

$$\begin{array}{r} 14 \\ 7\overline{)100} \\ \underline{98} \\ 2 \end{array}$$

Last three digit number divisible by 7 = 994.

999 − 5 = 994

$$\begin{array}{r} 142 \\ 7\overline{)999} \\ \underline{98} \\ 19 \\ \underline{14} \\ 5 \end{array}$$

$\therefore$ The three digit number divisible by 7 are :

105, 112, 119 ... 994

Here $a = 105$, $d = 112 - 105 = 7$

$a_n = 994 \quad n = ?$

$a_n = a + (n - 1)\, d$

$\therefore\ 994 = 105 + (n - 1)7$

$\therefore\ 994 = 105 + 7n - 7$

$\therefore\ 994 = 98 + 7n$

$\therefore\ 994 - 98 = 7n$

$\therefore\ 896 = 7n$

$\therefore\ n = 128$

Thus, the three digit numbers divisible by 7 are 128.

13. The sum of the 4^{th} and 8^{th} terms of an AP is 24 and the sum of the 6^{th} and 10th terms is 44. Find the first three terms of the AP.

➤ Let first term = a and

the common difference = d

$a_4 + a_8 = 24$

$\therefore\ (a + 3d) + (a + 7d) = 24$

$\therefore\ a + 3d + a + 7d = 24$

$\therefore\ 2a + 10d = 24$

$\therefore\ a + 5d = 12$ ....(i)

$a_6 + a_{10} = 44$

$(a + 5d) + (a + 9d) = 44$

$a + 5d + a + 9d = 44$

$\therefore\ 2a + 14d = 44$

$\therefore\ a + 7d = 22$ ...(ii)

(ii) Subtracting equation (i) from (ii) we get

$\therefore\ (a + 7d) - (a + 5d) = 22 - 12$

$\therefore\ a + 7d - a - 5d = 22 - 12$

$\therefore\ 2d = 10$

$\therefore\ d = 5$

➤ Putting $d = 5$ in eq. (i)

$a + 5d = 12$

$\therefore\ a + (5 \times 5) = 12$

$\therefore\ a + 25 = 12$

$\therefore\ a = -13$

➤ An AP : $a, a + d, a + 2d, a + 3d, a + 4d, \ldots$

$a = -13$

$a + d = -13 + 5 = -8$

$a + 2d = -13 + (2 \times 5) = -13 + 10 = -3$

Thus, the first three terms of the AP are

$-13, -8$ and -3

14. How many terms of the A.P : 9, 17, 25, ... must be taken to give a sum of 636 ?

➤ Here, $a = 9$, $d = 17 - 9 = 8$,

Let the sum of n^{th} term is 636

$\therefore\ S_n = 636$

Now, $S_n = \frac{n}{2}\ [2a + (n - 1)d]$

$\therefore\ 636 = \frac{n}{2}\ [2 \times 9 + (n - 1)8]$

$\therefore\ 636 = \frac{n}{2} \times 2\ [9 + (n - 1)4]$

$\therefore\ 636 = n\ [9 + 4n - 4]$

$\therefore\ 636 = n\ [5 + 4n]$

$\therefore\ 636 = 5n + 4n^2$

$4n^2 + 5n - 636 = 0$

$4n^2 + 53n - 48n - 636 = 0$

$\therefore\ n(4n + 53) - 12\ (4n + 53) = 0$

$\therefore\ (4n + 53)\ (n - 12) = 0$

$4n + 53 = 0$ or $n - 12 = 0$

$\therefore\ n = \frac{-53}{4}$ or $n = 12$

n is never negative or fraction

So, $n = \frac{-53}{4}$ is rejected so, the sum of first 12 terms is 636.

15. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

➤ Here, $n = 51$, $a_2 = 14$, $a_3 = 18$

Let the first term of an AP is a and its common difference is d.

➤ $a_2 = 14$ and $a_3 = 18$

$\therefore a_2 = a + d$	$\therefore a_3 = a + 2d$
$\therefore a + d = 14$ ...(i)	$\therefore a + 2d = 18$ ....(ii)

➤ Subtracting equation (ii) from (i) we have

$18 - 14 = (a + 2d) - (a + d)$

$\therefore 4 = a + 2d - a - d$

$\therefore 4 = d$

➤ $a + d = 14$

$\therefore a + 4 = 14$

$\therefore a = 14 - 4$

$\therefore a = 10$

➤ $S_n = \frac{n}{2}\ [2a + (n - 1)d$

Taking $n = 51$, we have

$$\therefore S_5 = \frac{51}{2}\ [2 \times 10 + (51 - 1)4]$$

$$= \frac{51}{2}\ [20 + (50)4]$$

$$= \frac{51}{2}\ [20 + 200]$$

$$= \frac{51}{2} \times 220$$

$\therefore S_{51} = 51 \times 110$

Thus, the sum of first 51 terms of an AP is 5610.

16. A sum of ₹ 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is ₹ 20 less than its preceding prize, find the value of each of the prizes.

➤ Second prize = ₹ $(a - 20)$

Third prize = (₹ $a - 40$)

Fourth prize = (₹ $a - 60$)

∞. Common difference d

$= (a - 20) - (a) = -20$

Number of prizes $n = 7$

and the sum of all prizes $= S_n = S_7 =$ ₹ 700

➤ $S_n = \frac{n}{2}\ [2a + (n - 1)d]$

$$\therefore 700 = \frac{7}{2}\ [2 \times a + (7 - 1) \times (-20)]$$

$$= \frac{7}{2}\ [2a + 6 \times (-20)]$$

$\therefore\ 700 = \frac{7}{2}\ [2a - 120]$

$700 \times 2\ = 7(2a - 120)$

$1400\ = 7\ (2a - 120)$

$\therefore\ \frac{1400}{7} = (2a - 120)$

$\therefore\ 200 = (2a - 120)$

$\therefore\ 200 + 120 = 2a$

$\therefore\ 320 = 2a$

$\therefore\ a = 160$

So, the first prize is a = ₹ 160 and the value of each of the prizes is ₹ 160, ₹ 140, ₹ 120, ₹ 100, ₹ 80, ₹ 60 and ₹ 40.

17. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students ?

➤ Total number of classes = 12

$\therefore\ n = 12$

There are three section of each class.

$\therefore$ Number of planting trees by class I = 1×3

$= 3$

$\therefore$ Number of planting trees by class II = 2×3

$= 6$

$\therefore$ Number of planting trees by class III = 3×3

$= 9$

$\therefore$ Number of planting trees by class IV = 4×3

$= 12$

and so on.

At last, Number of planting trees by class XII = 12×3

$= 36$

So, we get the following terms 3, 6, 12, 15 16

Which are in AP First term a = 3

Common difference d = 6 – 3= 3

Now, $S_n = \frac{n}{2}\ [a + a_n]$

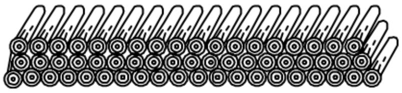
$\therefore\ S_{12} = \frac{12}{2}\ [3 + 36]$

$= 6\ [39]$

$= 234$

So, 234 trees will be planted by the students.

18. 200 logs are stacked in the following manner : 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see Fig.). In how many rows are the 200 logs placed and how may logs are in the top now ?



➤ we have,

20 logs in the bottom row

19 logs in the next row

18 logs in the row next to it and so on.

Total number of logs = 200

So, the terms are :

20, 19, 18, ...

which are in AP

$a = 20$, $d = 19 - 20 = -1$, and $S_n = 200$, $n = ?$

➤ $$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$200 = \frac{n}{2}[2 \times 20 + (n-1)(-1)]$$

$\therefore 200 \times 2 = n[40 - n + 1]$

$\therefore 400 = n[-n + 41]$

$\therefore 400 = -n^2 + 41n$

$\therefore n^2 - 41n + 400 = 0$

$\therefore n^2 - 25n - 16n + 400 = 0$

$\therefore n(n-25) - 16(n-25) = 0$

$\therefore (n-25)(n-16) = 0$

$\therefore n - 25 = 0$ or $n - 16 = 0$

$\therefore n = 25$or $n = 16$

➤ If $n = 25$ then the number of logs in the top row is,

$a_n = a + (n-1)d$

$\therefore a_{25} = 20 + (25-1)(-1)$

$= 20 - 24 = -4$ which is rejected.

If $n = 16$ then the number of logs in the top row is

$a_{16} = a + (16-1)d$

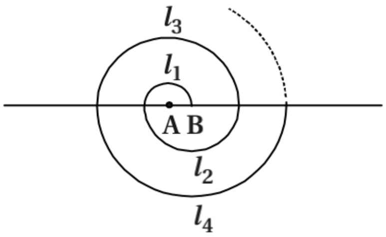
$= 20 + 15(-1)$

$= 20 - 15$

$= 5$

So, there are 5 logs in the top row and the 200 logs are placed in 16 rows

19. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 2.0 cm, ... as shown in Fig. 5.4. What is the total length of such a spiral made up of thirteen consecutive semicircles ? $\left(\text{Take}\,\pi = \frac{22}{7}\right)$



[Hints : Length of successive semicircles is l_1, l_2, l_3, l_4, ··· with centres at A, B, A, B respectively.

Length of each semicircle = πr.

➤ Length of first semicircle $l_1 = 0.5\ \pi$ cm $1 \times 0.5\ \pi$ cm

➤ Length of second semicircle $l_2 = 1.0\ \pi$ cm = $2 \times 0.5\ \pi$ cm

➤ Length of third semicircle $l_3 = 1.5\ \pi$ cm = $3 \times 0.5\ \pi$ cm

and so on.

➤ The length of 4th semicircle

$l_4 = 2.0\ \pi$ cm = $4 \times 0.5\ \pi$ cm

➤ At last, length of 13 the semicircle $l_3 = 13 \times 0.5$ cm

So, the total length of such a spiral made up of thirteen consecutive semicircles.

$= l_1 + l_2 + l_3 + l_4 + l_5 \ldots\ldots\ldots + l_{13}$

$= 0.5\pi\ [1 + 2 + 3 + 4 + 5 + \ldots + 13]$ cm ....(i)

➤ Now, $1 + 2 + 3 + \ldots. + 13$ are in AP

Here, first term $a = 1$

and common difference $d = 1$

$n = 13$

Last term $a_n = 13$

Now, $S_n = \frac{n}{2}\ [a + a_n]$

$\therefore\ S_{13} = \frac{13}{2}\ (1 + 13)$

$= \frac{13}{2}\ (14)$

$= 13 \times 7$

$\therefore\ S_{13} = 91$

From (1), we have

The required total length of spiral

$= 0.5\ \pi \times 91$

$= \frac{5}{10} \times \frac{22}{7} \times 91$

$= 11 \times 13$

$= 143$ cm

20. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

➤ Let the first term of an AP is a and its common difference is d.

Given that $S_7 = 49$ and $S_{17} = 289$

$S_7 = 49$

Now, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore\ 49 = \frac{7}{2}[2a + (7 - 1)d]$

$\therefore\ 49 \times 2 = 7\,[2a + 6d]$

$\therefore\ 98 = 7\,[2a + 6d]$

$\therefore\ \frac{98}{7} = 2a + 6d$

$\therefore\ 14 = 2a + 6d$

$\therefore\ 7 = a + 3d$

$\therefore\ a + 3d = 7$ ....(i)

$S_{17} = 289$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore\ 289 = \frac{17}{2}[2a + (17 - 1)\ d]$

$\therefore\ 289 \times 2 = 17\,[2a + 16d]$

$\therefore\ 578 = 17\,[2a + 16d]$

$\therefore\ \frac{578}{17} = 2a + 16d$

$\therefore\ \frac{578}{17} = 2(a + 8d)$

$\therefore\ \frac{578}{34} = a + 8d$

$\therefore\ 17 = a + 8d$ ....(ii)

➤ $a + 3d = 7$ ...(i)

$a + 8d = 17$ ...(ii)

$-5d = -10$

$\therefore\ 5d = 10$

$\therefore\ d = 2$

➤ $a + 3d = 7$

$\therefore\ a + 3 \times 2 = 7$

$\therefore\ a + 6 = 7$

$\therefore\ a = 7 - 6$

$\therefore\ a = 1$

➤ $a = 1$ and $d = 2$, $S_n = ?$

➤ Now, $S_n = \frac{n}{2}\ [2a + (n - 1)d]$

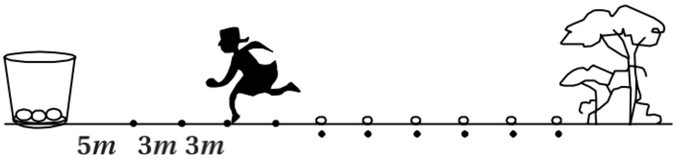
$\therefore\ S_n = \frac{n}{2}\ [2 \times 1 + (n - 1)2]$

$= \frac{n}{2}\ [2 + 2n - 2]$

$= \frac{n}{2}\ [2n]$

$\therefore\ S_n = n^2$

21. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (see Fig.).



A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

[Hint : To pick up the first potato and the second potato, the total distance (in metres) run by a competitor is $2 \times 5 + 2 \times (5 + 3)$]

➤ Distance to run for first potato $5 \times 2 = 10$ m

Distance to run for second potato $= (5 + 3) \times 2 = 16$ m

Distance to run for third potato $= (5 + 3 + 3) \times 2 = 22$ m

Distance to run for fourth potato $= (5 + 3 + 3 + 3) \times 2 = 28$ cm

The terms are 10, 16, 22, 28, ...

Which are in AP

$a = 10,\ d = 16 - 10 = 6$

There are 10 potatoes in the line $n = 10$

$$S_n = \frac{n}{2}\ [2a + (n - 1)\ d]$$

$$\therefore\ S_{10} = \frac{10}{2}\ [2(10) + (10 - 1) \times 6)$$

$$= 5[20 + 54]$$

$$= 5[74]$$

$$= 370$$

So, The total distance the competitor has to run is 370 m.

22. Ramkali saved ₹ 5 in the first week of a year and then increased her weekly savings by ₹ 1.75. If in the nth week, her weekly savings become ₹ 20.75, find n.

➤ $a = 5$ ₹, $d = 1.75$ ₹, $a_n = 20.70$ ₹, $n = ?$

$$a_n = a + (n - 1)d$$

$$\therefore\ 20.75 = 5 + (n - 1)\ 1.75$$

$$\therefore\ 20.75 = 5 + 1.75n - 1.75$$

$$\therefore\ 20.75 = 1.75n + 3.25$$

$$\therefore\ 20.75 - 3.25 = 1.75n$$

$$\therefore\ 17.5 = 1.75n$$

$$\therefore\ n = 10$$

23. Find the 20th term from the last term of the AP : 3, 8, 13, ..., 253.

➤ Write the terms of the given AP in reverse, we have 253... 13, 8, 3.

First term $a = 253$

Common difference $d = 8 - 13 = -5$

we have to find 20th term for this

$\therefore\ a_n = a + (n - 1)d$

$\therefore\ a_{20} = 253 + (20 - 1)\ (-5)$

$= 253 + (19)(-5)$

$= 253 - 95$

$= 158$

Thus, the 20th term from the last term of the given AP is 158.

24. Two APs have the same common difference. The difference between their 100th terms is 100, what is the difference between their 1000th terms ?

➤ Let the first term of first AP is a_1 and the first term and second AP is a_2 $(a_1 > a_2)$

Given that they have same common difference.

100th terms of first AP is

$a_{100} = a_1 + (100 - 1)d$

$\therefore\ a_{100} = a_1 + 99d$ ....(i)

and 100th term of second AP is

$a'_{100} = a_2 + (100 - 1)d$

$= a_2 + 99d$ ....(ii)

Given that $a_{100} - a'100 = 100$

$\therefore$ From equation (i) and (ii)

$\therefore\ a_1 + 99d - (a_2 + 99d) = 100$

$\therefore\ a_1 - a_2 = 100$ ....(iii)

➤ 1000th term of first A.P is

$a_{1000} = a_1 + (1000 - 1)\ d$

$= a_1 + 999d$ ....(iv)

➤ 1000th term m is second AP is

$a'_{1000} = a_2 + (1000 - 1)d$

$= a_2 + 999d$ ....(v)

➤ From equation (iv) and (v),

$a_{1000} - a'_{1000} = (a_1 + 999d) - (a_2 + 999d)$

$= a_1 - a_2$

$= 100$ (From equation (iii))

Thus, the difference between their 1000th term is 100.

25. In an AP : Given $a = 8$, $a_n = 62$ and $S_n = 210$ find n and d.

➤ Here, $a = 8$, $a_n = 62 = l$, $S_n = 210$, $n = ?$, $d = ?$

Now, $S_n = \frac{n}{2}\ (a + l)$

$\therefore\ 210 = \frac{n}{2}\ (8 + 62)$

$= \frac{n}{2}\ (70)$

$210 = 35n$

$\therefore\ n = 6$

➤ $a_n = 62,\ a = 8,\ n = 6,\ d = ?$

$a_n = a + (n - 1)d$

$\therefore\ 62 = 8 + (6 - 1)d$

$\therefore\ 62 = 8 + (5)d$

$\therefore\ 62 = 8 + 5d$

$\therefore\ 62 - 8 = 5d$

$\therefore\ 54 = 5d$

$\therefore\ d = \frac{54}{5}$

Thus, $n = 6$ and $d = \frac{54}{5}$.

26. Check whether –150 is a term of the AP : 11, 8, 5, 2 ... First term $a = 11$

➤ Common difference $d = 8 - 11$

$= -3$

Let the n^{th} term of the given A. P. is –150

$\therefore\ a_n = -150$

➤ $an = a + (n - 1)\ d$

$\therefore\ (-150) = 11 + (n - 1)\ (-3)$

$\therefore\ (-150) = 11 + (-3n + 3)$

$\therefore\ (-150) = 11 - 3n + 3$

$\therefore\ -150 = 14 - 3n$

$\therefore\ -150 - 14 = -3n$

$\therefore\ -164 = -3n$

$\therefore\ 3n = 164$

$\therefore\ 3n = 164$

$\therefore\ n = 54\ \frac{2}{3}$

But n should be a positive interger. Here, n is a positive fraction.

So, (–150) is not a term of the given list of numbers.