OPEN STUDENT FOUNDATION
STD 10 : MATHS
IMPORTANT QUESTION DAY 3

Section A

- **Write the answer of the following questions. [Each carries 1 Mark]** **[3]**

1. If roots of the equation $x^2 - kx + 4 = 0$ are equal then k = ... (4 –4, ±4)
2. Roots of the quadratic equation are equal then $b^2 - 4ac$ = ... (1, 2, 0)
3. Comparing the equation $5x^2 + 3x - 8 = 2x$ with the standard form of the quadratic equation then b = ...

Section B

- **Write the answer of the following questions. [Each carries 2 Marks]** **[8]**

4. Check whether the following is quadratic equations : $(2x - 1)(x - 3) = (x + 5)(x - 1)$
5. Check whether the following is quadratic equations : $x^3 - 4x^2 - x + 1 = (x - 2)^3$
6. Solve the problem given in Example (1) : $x^2 - 55x + 750 = 0$
7. Find the root of the following quadratic equation by factorisation : $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

Section C

- **Write the answer of the following questions. [Each carries 3 Marks]** **[24]**

8. Represent the following situations in the form of quadratic equations : The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.
9. Is the following situation possible ? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.
10. Find the value of k of the following quadratic equations, so that they have two equal roots. $kx(x - 2) + 6 = 0$
11. Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2 ? If so, find its length and breadth.
12. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them : $3x^2 - 4\sqrt{3}x + 4 = 0$
13. Find two numbers whose sum is 27 and product is 182.
14. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.
15. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was ₹ 90, find the number of articles produced and the cost of each article.

Section A

● **Write the answer of the following questions. [Each carries 1 Mark]** **[3]**

1. If roots of the equation $x^2 - kx + 4 = 0$ are equal then $k = ...$ (4 –4, ±4)

➤ Roots of the equation $x^2 - kx + 4 = 0$ are equal then $b^2 - 4ac = 0$

$b^2 - 4ac = 0$

$\therefore (-k)^2 - 4(1)(4) = 0$

$\therefore k^2 - 16 = 0$

$\therefore k^2 = 16$

$\therefore k = \pm 4$

2. Roots of the quadratic equation are equal then $b^2 - 4ac = ...$ (1, 2, 0)

➤ Roots are equal then $b^2 - 4ac = 0$.

3. Comparing the equation $5x^2 + 3x - 8 = 2x$ with the standard form of the quadratic equation then $b = ...$

➤ $5x^2 + 3x - 8 = 2x$

$\therefore 5x^2 + 3x - 8 - 2x = 0$

$5x^2 + x - 8 = 0$

With $ax^2 + bx + c = 0$

we have $a = 5, b = 1, c = -8$

Hence $b = 1$

Section B

● **Write the answer of the following questions. [Each carries 2 Marks]** **[8]**

4. Check whether the following is quadratic equations : $(2x - 1)(x - 3) = (x + 5)(x - 1)$

➤ $(2x - 1)(x - 3) = (x + 5)(x - 1)$ ($\because$ Expanding)

$\therefore 2x^2 - 6x - x + 3 = x^2 - x + 5x - 5$

$\therefore 2x^2 - 6x - x + 3 - x^2 + x - 5x + 5 = 0$

Thus, $x^2 - 11x + 8$ is a quadratic polynomial

$\therefore x^2 - 11x + 8 = 0$ is a quadratic equation

5. Check whether the following is quadratic equations : $x^3 - 4x^2 - x + 1 = (x - 2)^3$

➤ $x^3 - 4x^2 - x + 1 = (x - 2)^3$

$\therefore x^3 - 4x^2 - x + 1 = x^3 + 3x^2(-2) + 3x(-2)^2 + (-2)^3$ ($\because$ Expanding)

$\therefore x^3 - 4x^2 - x + 1 = x^3 - 6x^2 + 3x(4) + (-8)$

$\therefore x^3 - 4x^2 - x + 1 = x^3 - 6x^2 + 12x - 8$

$\therefore\ x^3 - 4x^2 - x + 1 - x^3 + 6x^2 - 12x + 8 = 0$

$\therefore\ 2x^2 - 13x + 9 = 0$ is a quadratic equation.

$\therefore\ x^3 - 4x^2 - x + 1 = (x - 2)^3$ is a quadratic equation

6. Solve the problem given in Example (1) : $x^2 - 55x + 750 = 0$

➤ $x^2 - 55x + 750 = 0$

$\therefore\ x^2 - (30 + 25)x + 750 = 0$

$\therefore\ x^2 - 30x - 25x + 750 = 0$

$\therefore\ x(x - 30) - 25(x - 30) = 0$

$\therefore\ (x - 30)\ (x - 25) = 0$

$\therefore\ x - 30 = 0$ or $x - 25 = 0$

$\therefore\ x = 30$ or $x = 25$

Hence, the roots of the equation

$x^2 - 55x + 750 = 0$ are 30 and 25.

7. Find the root of the following quadratic equation by factorisation : $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

➤ $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\therefore\ \sqrt{2}x^2 + (2 + 5)x + 5\sqrt{2} = 0$

$\therefore\ \sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$

$\therefore\ \sqrt{2}x(x + \sqrt{2}) + 5(x + \sqrt{2}) = 0$

$\therefore\ (x + \sqrt{2})\ (\sqrt{2}x + 5) = 0$

$\therefore\ x + \sqrt{2} = 0$ or $\sqrt{2}x + 5 = 0$

$\therefore\ x = -\sqrt{2}$ or $x = -\dfrac{5}{\sqrt{2}}$

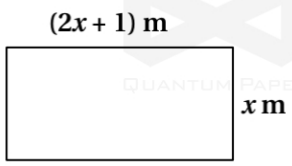
Hence, $-\sqrt{2}$ and $\dfrac{-5}{\sqrt{2}}$ are the roots of the given equation $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$.

Section C

● **Write the answer of the following questions. [Each carries 3 Marks]** **[24]**

8. Represent the following situations in the form of quadratic equations : The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

➤ Let the breadth of a rectangular plot = x m



The length of the plot is one more than twice its breadth.

length of the plot is one more than twice its breadth.

$\therefore$ length of the plot = $(2x + 1)$ m

Area of a rectangular plot $= \text{length} \times \text{breadth}$

$$= (2x + 1)x$$

$$528 = 2x^2 + x$$

$$\therefore\ 528 = 2x^2 + x$$

$$\therefore\ 2x^2 + x - 528 = 0$$

Hence, the required quadratic equation is $2x^2 + x - 528 = 0$

9. Is the following situation possible ? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

➤ Let the present age of one friend is x years.

Give that the sum of the ages of two friend is 20 years.

So, the age of second friend is $(20 - x)$ years

➤ Four years ago,

the age of one friend $= (x - 4)$ years and the age of second friend $= (20 - x - 4)$

$= (16 - x)$ years

➤ Four years age, the product of their age is 48.

$\therefore\ (x - 4)\ (16 - x) = 48$

$\therefore\ 16x - x^2 - 64 + 4x = 48$

$\therefore\ 20x - x^2 - 64 - 48 = 0$

$\therefore\ x^2 - 20x - 112 = 0$

➤ Comparing this equation with

$ax^2 + bx + c = 0$, we have $a = 1$, $b = -20$ and $c = 112$

➤ Discriminant $D = b^2 - 4ac$

$= (-20)^2 - 4(1)(112)$

$= 400 - 448$

$= -48 < 0$

➤ So, $D < 0$ the equation has no real roots.

Hence, we can not find the age of two friends.

$\therefore$ The given situation is not possible.

10. Find the value of k of the following quadratic equations, so that they have two equal roots. $kx(x - 2) + 6 = 0$

➤ $kx(x - 2) + 6 = 0$

$\therefore\ kx^2 - 2kx + 6 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we have

$a = k$, $b = -2k$ and $c = 6$

➤ Given that the equation has equal roots.

$\therefore\ D = 0$

$\therefore\ b^2 - 4ac = 0$

$\therefore\ (-2k)^2 - 4(k)(6) = 0$

$\therefore\ 4k^2 - 24k = 0$

$\therefore\ k^2 - 6k = 0$

$\therefore\ k(k - 6) = 0$

$\therefore\ k = 0$ or $k - 6 = 0$

$\therefore\ k = 0$ or $k = 6$

But $k \neq 0$ as in the quadratic equation

$ax^2 + bx + c = 0,\ a \neq 0$

$\therefore\ k = 6$

11. Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2 ? If so, find its length and breadth.

➤ Length the length of a rectangular park be x m and its width be y m.

Perimeter of the rectangle = 80 m

$\therefore$ 2(Length + Breadth) = 80 m

$\therefore\ x + y = 40$

$\therefore\ y = 40 - x$

➤ The area of rectangular park = 400 m^2

$\therefore\ xy = 400$

$\therefore\ x(40 - x) = 400\ (\because\ y = 40 - x)$

$\therefore\ 40x - x^2 = 400$

$\therefore\ 40x - x^2 - 400 = 0$

➤ $x^2 - 40x + 400 = 0$

comparing this equation with $ax^2 + bx + c = 0$

$a = 1,\ b = -40$ and $c = 400$

➤ Discriminant D $= b^2 - 4ac$

$= (-40)^2 - 4(1)(400)$

$= 1600 - 1600$

$= 0$

➤ Here D = 0,

so the equation has two equal real roots

Roots $x = \dfrac{-b}{2a}$

$= \dfrac{-(-40)}{2 \times 1} = 20$

➤ $\therefore$ Length $x = 20\ m$ and breadth $y = 40 - x$

$= 40 - 20$

$= 20$ m

This, it is possible to design a rectangular park.

12. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them : $3x^2 - 4\sqrt{3}x + 4 = 0$

➤ Comparing the equation $3x^2 - 4\sqrt{3}x + 4 = 0$

with $ax^2 + bx + c = 0$ we have

$a = 3,\ b = -4\sqrt{3},\ c = 4$

➤ Discriminant D $= b^2 - 4ac$

$= (-4\sqrt{3})^2 - 4(3)(4)$

$= (16 \times 3) - 48$

$= 48 - 48$

$= 0$

➤ Here, D = 0 so the equation has two equal real roots.

$$x = \frac{-b}{2a}$$

$$= \frac{-(-4\sqrt{3})}{2 \times 3}$$

$$= \frac{4\sqrt{3}}{2\sqrt{3} \times \sqrt{3}}$$

$$= \frac{4}{2\sqrt{3}}$$

$$= \frac{2}{\sqrt{3}}$$

Hence, the equal roots are $\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$.

13. Find two numbers whose sum is 27 and product is 182.

➤ Let one number be x and product of two number is 182.

So, the other number is $\frac{182}{x}$

➤ Also the sum of two numbers is 27

$\therefore x + \frac{182}{x} = 27$

$\therefore x^2 + 182 = 27x$

$\therefore x^2 - 27x + 182 = 0$

$\therefore x^2 - (14 + 13)x + 182 = 0$

$\therefore x^2 - 14x - 13x + 182 = 0$

$\therefore x(x - 14) - 13(x - 14) = 0$

$\therefore (x - 14)\ (x - 13) = 0$

$\therefore x - 14 = 0$ or $x - 13 = 0$

$\therefore\ x = 14$ or $x = 13$

Thus, If one number is 14 the other number is $\frac{182}{14} = 13$.

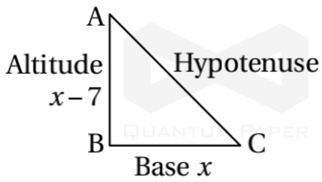
Or If one number is 13 then the other number is $\frac{182}{13} = 14$.

Verification :

The sum of 14 and 13 is 27

and their product is $13 \times 14 = 182$.

14. The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

➤ 

➤ In a right angle triangle

Let the base is x cm

Its altitude is 7 cm less than its base.

$\therefore$ Altitude = $(x - 7)$ cm.

$AB^2 + BC^2 = AC^2$

$\therefore\ (x-7)^2 + x^2 = (13)^2$

$\therefore\ x^2 - 14x + 49 + x^2 = 169$

$\therefore\ 2x^2 - 14x + 49 - 169 = 0$

$\therefore\ 2x^2 - 14x - 120 = 0$

$\therefore\ x^2 - 7x - 60 = 0$ ($\because$ divide by 2)

$\therefore\ (x - 12)(x + 5) = 0$ ($\because$ factors)

$\therefore\ x - 12 = 0$, or $x + 5 = 0$

$\therefore\ x = 12$ or $x = -5$ (Impossible)

➤ $\therefore$ Length of the base x = 12 cm

➤ and Length of altitude = $x - 7 = 12 - 7 = 5$ cm

15. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was ₹ 90, find the number of articles produced and the cost of each article.

➤ Let in a cottage industry, the number of articles produce in a day is x

The cost of production of each article = ₹ $(2x + 3)$

Total cost of production = ₹ 90

➤ $x(2x + 3) = 90$

$\therefore\ 2x^2 + 3x - 90 = 0$

$\therefore\ 2x^2 + (15 - 12)x - 90 = 0$

$\therefore\ 2x^2 + 15x - 12x - 90 = 0$

$\therefore\ x(2x + 15) - 6(2x + 15) = 0$

$\therefore\ (2x + 15)(x - 6) = 0$

$\therefore\ 2x + 15 = 0$ or $x - 6 = 0$

$\therefore\ 2x = -15$ (અશક્ય) or $x = 6$

$\therefore\ x = 6$ or $x = \frac{-15}{2}$

➤ But $x = \frac{-15}{2}$ is not negative

$\therefore\ x = 6$

$\therefore$ The number of articles $x = 6$

and the cost of each article $= 2x + 3$

$= 12 + 3$

$=$ ₹ 15

➤ Thus, the number of articles = 6, cost of each article = ₹ 15