OPEN STUDENT FOUNDATION
Std 12 : MATHS
PRACTICE SHEET DAY 10

Section A

- **Write the answer of the following questions. [Each carries 2 Marks]** **[6]**

1. If a line makes angles 90°, 135°, 45° with the x, y and z- axes respectively, find its direction cosines.
2. Show that the line through the points (4, 7, 8), (2, 3, 4) is parallel to the line through the points (–1, –2, 1), (1, 2, 5).
3. Find the cartesian equation of the line which passes through the point (–2, 4, –5) and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$.

Section B

- **Write the answer of the following questions. [Each carries 3 Marks]** **[18]**

4. Show that the points (2, 3, 4), (– 1, – 2, 1), (5, 8, 7) are collinear.
5. Find the angle between the pair of lines :
$\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ and $\frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$
6. Find the angle between the pairs of lines :
$\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ and $\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$
7. Find the shortest distance between the lines $\vec{r} = \left(\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$ and $\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$.
8. Find the vector equation of the line passing through the point (1, 2, –4) and perpendicular to the two lines : $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.
9. Find the shortest distance between lines $\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$ and $\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$.

Section C

- **Write the answer of the following questions. [Each carries 4 Marks]** **[12]**

10. Find the direction cosines of the sides of the triangle whose vertices are (3, 5, – 4), (– 1, 1, 2) and (– 5, – 5, – 2).
11. Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$.
12. Find the shortest distance between the lines whose vector equations are
$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$.

Section A

- **Write the answer of the following questions. [Each carries 2 Marks]** **[6]**

1. If a line makes angles 90°, 135°, 45° with the x, y and z- axes respectively, find its direction cosines.

➥ Given that $\alpha = 90°$, $\beta = 135°$ and $\gamma = 45°$

$\therefore \cos\alpha = \cos 90° = 0,$

$\cos\beta = \cos 135° = \cos(180° - 45°)$

$= -\cos 45°$

$= -\frac{1}{\sqrt{2}}$

and $\cos\gamma = \cos 45° = \frac{1}{\sqrt{2}}$

$\therefore$ Hence, the direction cosines of the given lines are $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$.

2. Show that the line through the points (4, 7, 8), (2, 3, 4) is parallel to the line through the points (–1, –2, 1), (1, 2, 5).

➥ Let the line L_1 passes through A(4, 7, 8) and B(2, 3, 4).

$\therefore$ Direction ratios of $\vec{AB}$, $a_1 = x_2 - x_1 = 2 - 4 = -2$

$b_1 = y_2 - y_1 = 3 - 7 = -4$

$c_1 = z_2 - z_1 = 4 - 8 = -4$

Let the line L_2 passes through C(–1, –2, 1) and D(1, 2, 5).

$\therefore$ Direction ratios of $\vec{CD}$, $a_2 = x_2 - x_1 = 1 + 1 = 2$

$b_2 = y_2 - y_1 = 2 + 2 = 4$

$c_2 = z_2 - z_1 = 5 - 1 = 4$

Now, $\frac{a_1}{a_2} = \frac{-2}{2} = -1$, $\frac{b_1}{b_2} = \frac{-4}{4} = -1$, $\frac{c_1}{c_2} = \frac{-4}{4} = -1$

$\therefore \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\therefore \vec{AB} \| \vec{CD}$

$\therefore$ Hence, the lines L_1 and L_2 are parallel.

3. Find the cartesian equation of the line which passes through the point (–2, 4, –5) and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$.

➥ The line L is passing through the point $A(\vec{a}) = (-2, 4, -5)$.

$\therefore (x_1, y_1, z_1) = (-2, 4, -5)$

The line L is parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$.

$\therefore$ Direction of the line L, $(a, b, c) = (3, 5, 6)$

$\therefore$ The cartesian equation of the line L :

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

$$\therefore \frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$$

Section B

- **Write the answer of the following questions. [Each carries 3 Marks]** **[18]**

4. Show that the points (2, 3, 4), (– 1, – 2, 1), (5, 8, 7) are collinear.

➥ Let A(2, 3, 4), B (–1, –2, 1) and C(5, 8, 7) are given points,

Direction ratios of AB,

$a_1 = x_2 - x_1 = -1 - 2 = -3$

$b_1 = y_2 - y_1 = -2 - 3 = -5$

$c_1 = z_2 - z_1 = 1 - 4 = -3$

Direction ratios of BC,

$a_2 = x_2 - x_1 = 5 + 1 = 6$

$b_2 = y_2 - y_1 = 8 + 2 = 10$

$c_2 = z_2 - z_1 = 7 - 1 = 6$

$$\text{Now } \left.\begin{array}{l} \frac{a_1}{a_2} = \frac{-3}{6} = -\frac{1}{2} \\ \frac{b_1}{b_2} = \frac{-5}{10} = -\frac{1}{2} \\ \frac{c_1}{c_2} = \frac{-3}{6} = -\frac{1}{2} \end{array}\right\} \Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\therefore$ AB || BC

AB and BC have common point B.

Hence, points A, B and C are collinear.

5. Find the angle between the pair of lines :

$\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ and $\frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$

➥ Line L_1 : $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$

Line L_2 : $\frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$

The vectors parallel to lines L_1 and L_2 are respectively.

$\vec{b_1} = 2\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b_2} = 4\hat{i} + \hat{j} + 8\hat{k}$ Let the angle between the two lines be θ then,

$\frac{\vec{b_1} \cdot \vec{b_2}}{}$

$\cos\theta = \dfrac{}{|\vec{b_1}|\,|\vec{b_2}|}$

$= \dfrac{(2\hat{i} + 2\hat{j} + \hat{k}) \cdot (4\hat{i} + \hat{j} + 8\hat{k})}{\sqrt{4 + 4 + 1}\,\sqrt{16 + 1 + 64}}$

$= \dfrac{8 + 2 + 8}{3 \times 9}$

$= \dfrac{18}{3 \times 9} = \dfrac{2}{3}$

$\therefore\ \theta = \cos^{-1}\left(\dfrac{2}{3}\right)$

6. Find the angle between the pairs of lines :

$\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ and $\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$

➥ Line L_1 : $\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda\,(3\hat{i} + 2\hat{j} + 6\hat{k})$

➥ Line L_2 : $\vec{r} = 7\hat{i} - 6\hat{k} + \mu\,(\hat{i} + 2\hat{j} + 2\hat{k})$

➥ Line L_1 is parallel to the vector $\vec{b_1} = 3\hat{i} + 2\hat{j} + 6\hat{k}$ and line L_2 is parallel to the vector $\vec{b_2} = \hat{i} + 2\hat{j} + 2\hat{k}$. Let the angle between the lines L_1 and L_2 be θ.

$\therefore\ \cos\theta = \dfrac{\vec{b_1} \cdot \vec{b_2}}{|\vec{b_1}|\,|\vec{b_2}|}$

$\therefore\ \cos\theta = \dfrac{(3\hat{i} + 2\hat{j} + 6\hat{k}) \cdot (\hat{i} + 2\hat{j} + 2\hat{k})}{\sqrt{9 + 4 + 36}\,\sqrt{1 + 4 + 4}}$

$= \dfrac{3 + 4 + 12}{(7)\,(3)}$

$= \dfrac{19}{21}$

$\therefore\ q = \cos^{-1}\left(\dfrac{19}{21}\right)$

7. Find the shortest distance between the lines $\vec{r} = \left(\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$ and $\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$.

➥ Line L_1 : $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$

Line L_1 is passing through the point $A_1(\vec{a_1}) = \hat{i} + 2\hat{j} + \hat{k}$ and it is parallel to the vector $\vec{b_1} = \hat{i} - \hat{j} + \hat{k}$.

Line L_2 : $\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu\,(2\hat{i} + \hat{j} + 2\hat{k})$

Line L_2 is passing through the point $A_2\,(\vec{a_2}) = 2\hat{i} - \hat{j} - \hat{k}$ and it is parallel to the vector $\vec{b_2} = 2\hat{i} + \hat{j} + 2\hat{k}$.

$\vec{a_2} - \vec{a_1} = (2\hat{i} - \hat{j} - \hat{k}) - (\hat{i} + 2\hat{j} + \hat{k})$

$= \hat{i} - 3\hat{j} - 2\hat{k}$

$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = -3\hat{i} - 0\hat{j} + 3\hat{k}$ $(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = (\hat{i} - 3\hat{j} - 2\hat{k}) \cdot (-3\hat{i} + 3\hat{k})$

$= -3 - 6 = -9$

$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{(-3)^2 + (3)^2} = 3\sqrt{2}$

$\therefore$ The shortest distance between the lines L_1 and L_2,

$$= \left|\frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|}\right|$$

$$= \left|\frac{-9}{3\sqrt{2}}\right| = \frac{3}{\sqrt{2}}$$

8. Find the vector equation of the line passing through the point (1, 2, –4) and perpendicular to the two lines : $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

➥ Line L_1 : $\frac{x - 8}{3} = \frac{y + 19}{-16} = \frac{z - 10}{7}$

Parallel vector to the line L_1 is

$\vec{b_1} = 3\hat{i} - 16\hat{j} + 7\hat{k}$

Line L_2 : $\frac{x - 15}{3} = \frac{y - 29}{8} = \frac{z - 5}{-5}$

Parallel vector to the line L_2 is $\vec{b_2} = 3\hat{i} + 8\hat{j} - 5\hat{k}$

Let the required line is parallel to the vector $\vec{b}$.

$\vec{b} = \vec{b_1} \times \vec{b_2}$

$= (3\hat{i} - 16\hat{j} + 7\hat{k}) \times (3\hat{i} + 8\hat{j} - 5\hat{k})$

$= 24\hat{i} + 36\hat{j} + 72\hat{k}$

The line L passes through A(1, 2, –4).

$\therefore \vec{a} = \hat{i} + 2\hat{j} - 4\hat{k}$

$\therefore$ The required equation of line L :

$\vec{r} = \vec{a} + \lambda\vec{b}, \lambda \in R$

$\therefore \vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda\,(24\hat{i} + 36\hat{j} + 72\hat{k})$

$\therefore \vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda\,(2\hat{i} + 3\hat{j} + 6\hat{k})$

9. Find the shortest distance between lines $\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k})$ and $\vec{r} = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$.

➥ Line L_1 : $\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda\,(\hat{i} - 2\hat{j} + 2\hat{k})$

Comparing with $r = a_1 + \lambda b_1$ we have,

$\vec{a_1} = 6\hat{i} + 2\hat{j} + 2\hat{k}$, $\vec{b_1} = \hat{i} - 2\hat{j} + 2\hat{k}$ Line L_2 : $\vec{r} = 4\hat{i} - \hat{k} + \mu\,(3\hat{i} - 2\hat{j} - 2\hat{k})$

Comparing with $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ we get,

$\vec{a_2} = -4\hat{i} - \hat{k}$ and $\hat{b_2} = 3\hat{i} - 2\hat{j} - 2\hat{k}$

$\vec{a_2} - \vec{a_1} = (-4\hat{i} - \hat{k}) - (6\hat{i} + 2\hat{j} + 2\hat{k})$

$= -10\hat{i} - 2\hat{j} - 3\hat{k}$

$$b_1 \times b_2 = \begin{vmatrix} i & j & k \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix} = 8\hat{i} + 8\hat{j} + 4\hat{k}$$

$|\vec{b_1} \times \vec{b_2}| = \sqrt{(8)^2 + (8)^2 + (4)^2}$

$= \sqrt{64 + 64 + 16} = 12$

The shortest distance between the lines L_1 and L_2

$$= \frac{|(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})|}{|\vec{b_1} \times \vec{b_2}|}$$

$$= \frac{|(-10\hat{i} - 2\hat{j} - 3\hat{k}) \cdot (8\hat{i} + 8\hat{j} + 4\hat{k})|}{12}$$

$$= \frac{|-80 - 16 - 12|}{12}$$

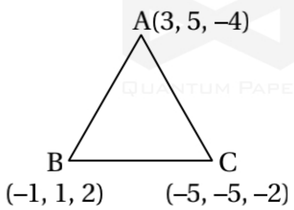
$$= \frac{108}{12} = 9$$

Section C

- **Write the answer of the following questions. [Each carries 4 Marks]** **[12]**

10. Find the direction cosines of the sides of the triangle whose vertices are (3, 5, – 4), (– 1, 1, 2) and (– 5, – 5, – 2).

➥ A(3, 5, –4), B(–1, 1, 2) and C(–5, –5, –2) are vertices of ΔABC.



AB = (–1, 1, 2) – (3, 5, –4)

= (–4, –4, 6)

BC = (–5, –5, –2) – (–1, 1, 2)

= (–4, –6, –4)

CA = (3, 5, –4) – (–5, –5, –2)

= (8, 10, –2)

Direction cosines of the side AB of ΔABC,

$$l = \frac{-4}{\sqrt{(-4)^2 + (-4)^2 + (6)^2}} = \frac{-4}{\sqrt{16+16+36}} = \frac{-4}{\sqrt{68}} = \frac{-4}{2\sqrt{17}} = \frac{-2}{\sqrt{17}}$$

$$m = \frac{-4}{\sqrt{(-4)^2 + (-4)^2 + (6)^2}} = \frac{-4}{\sqrt{16+16+36}} = \frac{-4}{\sqrt{68}} = \frac{-4}{2\sqrt{17}} = \frac{-2}{\sqrt{17}}$$

$$n = \frac{6}{\sqrt{(-4)^2 + (-4)^2 + (6)^2}} = \frac{6}{\sqrt{16+16+36}} = \frac{6}{\sqrt{68}} = \frac{6}{2\sqrt{17}} = \frac{3}{\sqrt{17}}$$

Direction cosines of the side BC of ΔABC

$$l = \frac{-4}{\sqrt{(-4)^2 + (-6)^2 + (-4)^2}} = \frac{-4}{\sqrt{16+36+16}} = \frac{-4}{\sqrt{68}} = \frac{-2}{\sqrt{17}}$$

$$m = \frac{-6}{\sqrt{(-4)^2 + (-6)^2 + (-4)^2}} = \frac{-6}{\sqrt{16+36+16}} = \frac{-6}{\sqrt{68}} = \frac{-3}{\sqrt{17}}$$

$$n = \frac{-4}{\sqrt{(-4)^2 + (-6)^2 + (-4)^2}} = \frac{-4}{\sqrt{16+36+16}} = \frac{-4}{\sqrt{68}} = \frac{-2}{\sqrt{17}}$$

Direction cosines of the side CA of ΔABC

$$l = \frac{8}{\sqrt{(8)^2 + (10)^2 + (-2)^2}} = \frac{8}{\sqrt{64+100+4}} = \frac{8}{\sqrt{168}} = \frac{4}{\sqrt{42}}$$

$$m = \frac{10}{\sqrt{(8)^2 + (10)^2 + (-2)^2}} = \frac{10}{\sqrt{64+100+4}} = \frac{10}{\sqrt{168}} = \frac{5}{\sqrt{42}}$$

$$n = \frac{-2}{\sqrt{(8)^2 + (10)^2 + (-2)^2}} = \frac{-2}{\sqrt{64+100+4}} = \frac{-2}{\sqrt{168}} = \frac{-1}{\sqrt{42}}$$

Hence, Direction cosines of $\vec{AB}$ $\frac{-2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{3}{\sqrt{17}}$

Direction cosines of $\vec{BC}$ $\frac{-2}{\sqrt{17}}, \frac{-3}{\sqrt{17}}, \frac{-2}{\sqrt{17}}$

Direction cosines of $\vec{CA}$ $\frac{4}{\sqrt{42}}, \frac{5}{\sqrt{42}}, \frac{-1}{\sqrt{42}}$

11. Find the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$.

➥ Line L_1 : $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$

$\Rightarrow \frac{x-(-1)}{7} = \frac{y-(-1)}{-6} = \frac{z-(-1)}{1}$

$\therefore$ Line L_1 passes through the point $A(\vec{a_1}) = -\hat{i} - \hat{j} - \hat{k}$ and it is parallel to the vector $\vec{b_1} = 7\hat{i} - 6\hat{j} + \hat{k}$.

Line L_2 : $\frac{x-3}{1} = \frac{z-5}{-2} = \frac{z-7}{1}$

$\therefore$ Line L_2 passes through the point $B(\vec{a_2}) = 3\hat{i} + 5\hat{j} + 7\hat{k}$ and it is parallel to the vector $\vec{b_2} = \hat{i} - 2\hat{j} + \hat{k}$.

$\vec{a_2} - \vec{a_1} = (3\hat{i} + 5\hat{j} + 7\hat{k}) - (-\hat{i} - \hat{j} - \hat{k})$

$$= 4\hat{i} + 6\hat{j} + 8\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix} = -4\hat{i} - 6\hat{j} - 8\hat{k}$$

$$|\vec{b_1} \times \vec{b_2}| = \sqrt{(-4)^2 + (-6)^2 + (-8)^2}$$

$$= \sqrt{16 + 36 + 64} = \sqrt{116} = 2\sqrt{29}$$

$$(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = (4\hat{i} + 6\hat{j} + 8\hat{k}) \cdot (-4\hat{i} - 6\hat{j} - 8\hat{k})$$

$$= -16 - 36 - 64 = -116$$

The shortest distance between the lines L_1 and L_2, $= \left| \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right|$

$$= \left| \frac{-116}{2\sqrt{29}} \right|$$

$$= \frac{116}{2\sqrt{29}} = 2\sqrt{29}$$

12. Find the shortest distance between the lines whose vector equations are $\vec{r} = (1 - t)\hat{i} + (t - 2)\hat{j} + (3 - 2t)\hat{k}$ and $\vec{r} = (s + 1)\hat{i} + (2s - 1)\hat{j} - (2s + 1)\hat{k}$.

➥ Line L_1 : $\vec{r} = (1 - t)\hat{i} + (t - 2)\hat{j} + (3 - 2t)\hat{k}$

$$\therefore \vec{r} = \hat{i} - 2\hat{j} + 3\hat{k} + t(-\hat{i} + \hat{j} - 2\hat{k})$$

Comparing with $\vec{r} = \vec{a_1} + t\vec{b_1}$ we have,

$$\vec{a_1} = \hat{i} - 2\hat{j} + 3\hat{k},\ \vec{b_1} = -\hat{i} + \hat{j} - 2\hat{k}$$

Line L_2 : $(s + 1)\hat{i} + (2s - 1)\hat{j} - (2s + 1)\hat{k} = (\hat{i} - \hat{j} - \hat{k}) + s(\hat{i} + 2\hat{j} - 2\hat{k})$

Comparing with $\vec{r} = \vec{a_2} + s\vec{b_2}$ we have,

$\vec{a_2} = \hat{i} - \hat{j} - \hat{k}$ and $\vec{b_2} = \hat{i} + 2\hat{j} - 2\hat{k}$

$$\vec{a_2} - \vec{a_1} = (\hat{i} - \hat{j} - \hat{k}) - (\hat{i} - 2\hat{j} + 3\hat{k})$$

$$= \hat{j} - 4\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix} = 2\hat{i} - 4\hat{j} - 3\hat{k}$$

$$|\vec{b_1} \times \vec{b_2}| = \sqrt{(2)^2 + (-4)^2 + (-3)^2}$$

$$= \sqrt{4 + 16 + 9} = \sqrt{29}$$

$$(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = (\hat{j} - 4\hat{k}) \cdot (2\hat{i} - 4\hat{j} - 3\hat{k})$$

$$= -4 + 12 = 8$$

The shortest distance between the lines L_1 and L_2,

$$= \left| \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

$$= \frac{8}{\sqrt{29}}$$