# OPEN STUDENT FOUNDATION
## Std 12 : MATHS
## PRACTICE SHEET DAY 9



### Section A

- **Write the answer of the following questions. [Each carries 2 Marks]** **[16]**

1. Find a vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units.
2. Show that the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-4\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.
3. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$.
4. If either vector $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$, then $\vec{a} \cdot \vec{b} = 0$. But the converse need not be true. Justify your answer with an example.
5. Show that, $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$.
6. Find $|\vec{a} \times \vec{b}|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.
7. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.
8. Show that the direction cosines of a vector equally inclined to the axes OX, OY and OZ are $\pm\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.

### Section B

- **Write the answer of the following questions. [Each carries 3 Marks]** **[15]**

9. Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$.
10. Show that the points A(1, 2, 7), B(2, 6, 3) and C(3, 10, –1) are collinear.
11. Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.
12. Find the area of the triangle with vertices A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5).
13. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $(2\vec{a} + \vec{b})$ and $(\vec{a} - 3\vec{b})$ externally in the ratio 1 : 2. Also, show that P is the mid point of the line segment RQ.

### Section C

- **Write the answer of the following questions. [Each carries 4 Marks]** **[20]**

14. Show that the points A, B and C with position vectors, $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$, respectively form the vertices of a right angled triangle.
15. Show that each of the given three vectors is a unit vector :
$\frac{1}{7}\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$, $\frac{1}{7}\left(3\hat{i} - 6\hat{j} + 2\hat{k}\right)$, $\frac{1}{7}\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)$.
Also, show that they are mutually perpendicular to each other.

16. The two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$. Find the unit vector parallel to its diagonal. Also, find its area.

17. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$.

18. The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of $\lambda$.

**Section A**

- **Write the answer of the following questions.** [Each carries 2 Marks] **[16]**

1. Find a vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units.

➥ $\vec{a} = 5\hat{i} - \hat{j} + 2\hat{k}$

$\therefore |\vec{a}| = \sqrt{(5)^2 + (-1)^2 + (2)^2}$

$= \sqrt{25 + 1 + 4} = \sqrt{30}$

The unit vector in the direction of $\vec{a}$

$\hat{a} = \frac{1}{|\vec{a}|}\vec{a}$

$= \frac{1}{\sqrt{30}}(5\hat{i} - \hat{j} + 2\hat{k})$

The vector with magnitude 8 units and in the direction of $\vec{a}$

$8\hat{a} = 8 \cdot \frac{1}{\sqrt{30}}(5\hat{i} - \hat{j} + 2\hat{k})$

$= \frac{40}{\sqrt{30}}\hat{i} - \frac{8}{\sqrt{30}}\hat{j} + \frac{16}{\sqrt{30}}\hat{k}$

2. Show that the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-4\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.

➥ Let $\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{B} = -4\hat{i} + 6\hat{j} - 8\hat{k}$

$\therefore$ The position vector A, $\vec{OA} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

$\therefore$ The position vector B, $\vec{OB} = -4\hat{i} + 6\hat{j} - 8\hat{k}$

$= -2(2\hat{i} - 3\hat{j} + 4\hat{k})$

$= -2\,\vec{OA}$

$\therefore$ $\vec{OA}$ and $\vec{OB}$ are parallel vector and their common point is O.

$\therefore$ $\vec{OA}$ and $\vec{OB}$ are collinear.

Hence the given vector are collinear.

3. Find the projection of the vector $\hat{i} + 3\hat{j} + 7\hat{k}$ on the vector $7\hat{i} - \hat{j} + 8\hat{k}$.

➥ $\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$ $\vec{b} = 7\hat{i} - \hat{j} + 8\hat{k}$

$\vec{a} \cdot \vec{b} = (\hat{i} + 3\hat{j} + 7\hat{k}) \cdot (7\hat{i} - \hat{j} + 8\hat{k})$

$= 7 - 3 + 56 = 60$

$|\vec{b}| = \sqrt{(7)^2 + (-1)^2 + (8)^2}$

$= \sqrt{49 + 1 + 64} = \sqrt{114}$

The projection of the vector $\vec{a}$ on the vector $\vec{b} = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \dfrac{60}{\sqrt{114}}$

**4.** If either vector $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$, then $\vec{a} \cdot \vec{b} = 0$. But the converse need not be true. Justify your answer with an example.

➡ Vector $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$ then $\vec{a} \cdot \vec{b} = \vec{0}$.

Its converse i.e. $\vec{a} \cdot \vec{b} = \vec{0}$,

$|\vec{a}|\,|\vec{b}| \cos\theta = 0$

But $|\vec{a}| \neq 0$, $|\vec{b}| \neq 0$ then $\cos\theta = 0$

So, $\theta = \dfrac{\pi}{2}$

$\therefore$ Vectors $\vec{a}$ and $\vec{b}$ are perpendicular.

Example,

$\vec{a} = 2\hat{i} + 5\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + 3\hat{k}$ then $\vec{a} \cdot \vec{b} = 4 - 10 + 6 = 0$. But $\vec{a} \neq 0$ and $\vec{b} \neq 0$.

The converse of the given statement is not true.

**5.** Show that, $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$.

➡ $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})$

$= \vec{a} \times (\vec{a} + \vec{b}) - \vec{b} \times (\vec{a} + \vec{b})$

$= \vec{a} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{a} - \vec{b} \times \vec{b}$

$= \vec{0} + \vec{a} \times \vec{b} + \vec{a} \times \vec{b} - \vec{0}$

$= 2(\vec{a} \times \vec{b})$

**6.** Find $|\vec{a} \times \vec{b}|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.

➡ $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

$$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{vmatrix}$$

$= \hat{i}(-14 + 14) - \hat{j}(2 - 21) + \hat{k}(-2 + 21)$

$= 19\hat{j} + 19\hat{k}$

$\therefore |\vec{a} \times \vec{b}| = \sqrt{(19)^2 + (19)^2}$

$= 19\sqrt{1+1} = 19\sqrt{2}$

**7.** If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

➥ $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$

$\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$

Now $2\vec{a} - \vec{b} + 3\vec{c}$

$= 2(\hat{i} + \hat{j} + \hat{k}) - (2\hat{i} - \hat{j} + 3\hat{k}) + 3(\hat{i} - 2\hat{j} + \hat{k})$

$= 3\hat{i} - 3\hat{j} + 2\hat{k}$

$\therefore$ The unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$

$= \dfrac{2\vec{a} - \vec{b} + 3\vec{c}}{|2\vec{a} - \vec{b} + 3\vec{c}|}$

$= \dfrac{3\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{9+9+4}} = \dfrac{3}{\sqrt{22}}\hat{i} - \dfrac{3}{\sqrt{22}}\hat{j} + \dfrac{2}{\sqrt{22}}\hat{k}$

**8.** Show that the direction cosines of a vector equally inclined to the axes OX, OY and OZ are $\pm\left(\dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}\right)$.

➥ The vector equally inclined to the axes OX, OY and OZ.

$\Rightarrow \alpha = \beta = \gamma$

Now $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\therefore \cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$

$\therefore 3\cos^2\alpha = 1$

$\therefore \cos\alpha = \dfrac{1}{\sqrt{3}}$

$\therefore \cos\alpha = \cos\beta = \cos\gamma = \dfrac{1}{\sqrt{3}}$

$\therefore$ The direction cosines of a required vector is $\dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}$.

## Section B

- **Write the answer of the following questions. [Each carries 3 Marks]** **[15]**

**9.** Find the angle between the vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$.

➥ $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$

$\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$

Now $|\vec{a}| = \sqrt{(1)^2 + (-2)^2 + (3)^2} = \sqrt{14}$

$$|\vec{b}| = \sqrt{(3)^2 + (-2)^2 + (1)^2} = \sqrt{14}$$

$$\vec{a} \cdot \vec{b} = (\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (3\hat{i} - 2\hat{j} + \hat{k})$$

$$= 3 + 4 + 3 = 10$$

Let the angle between the vectors $\vec{a}$ and $\vec{b}$ is $\theta$.

$$\therefore \cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

$$\therefore \cos\theta = \frac{10}{\sqrt{14}\sqrt{14}}$$

$$\therefore \cos\theta = \frac{10}{14}$$

$$\therefore \theta = \cos^{-1}\left(\frac{5}{7}\right)$$

$\therefore$ The angle between the given vectors is $\cos^{-1}\left(\frac{5}{7}\right)$.

**10.** Show that the points A(1, 2, 7), B(2, 6, 3) and C(3, 10, –1) are collinear.

➥ A(1, 2, 7), B(2, 6, 3) and C(3, 10, –1) Given points.

$$\therefore \vec{OA} = \hat{i} + 2\hat{j} + 7\hat{k}$$

$$\vec{OB} = 2\hat{i} + 6\hat{j} + 3\hat{k}$$

$$\vec{OC} = 3\hat{i} + 10\hat{j} - \hat{k}$$

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$= (2\hat{i} + 6\hat{j} + 3\hat{k}) - (\hat{i} + 2\hat{j} + 7\hat{k})$$

$$= \hat{i} + 4\hat{j} - 4\hat{k}$$

$$\vec{BC} = \vec{OC} - \vec{OB}$$

$$= (3\hat{i} + 10\hat{j} - \hat{k}) - (2\hat{i} + 6\hat{j} + 3\hat{k})$$

$$= \hat{i} + 4\hat{j} - 4\hat{k}$$

$$\vec{CA} = \vec{OA} - \vec{OC}$$

$$= (\hat{i} + 2\hat{j} + 7\hat{k}) - (3\hat{i} + 10\hat{j} - \hat{k})$$

$$= -2\hat{i} - 8\hat{j} + 8\hat{k}$$

$$= -2(\hat{i} + 4\hat{j} - 4\hat{k})$$

$$= -2\vec{BC}$$

Here $\vec{CA} \parallel \vec{BC}$ and point C is their common point.

Also $\vec{AB} = \vec{BC}$

$\therefore$ Hence, the points A, B, C are collinear.

**11.** Find the area of the parallelogram whose adjacent sides are determined by the vectors $\vec{a} = \vec{i} - \vec{j} + 3\vec{k}$ and $\vec{b} = 2\vec{i} - 7\vec{j} + \vec{k}$.

➥ The two adjacent sides of a parallelogram are defermined by the vector $\vec{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 7\hat{j} + \hat{k}$.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{vmatrix}$$

$$= \hat{i}(-1+21) - \hat{j}(1-6) + \hat{k}(-7+2)$$

$$= 20\hat{i} + 5\hat{j} - 5\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{(20)^2 + (5)^2 + (-5)^2}$$

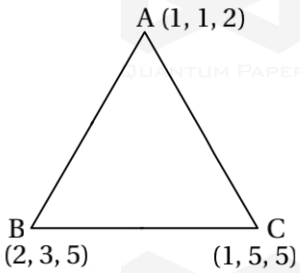
$$= \sqrt{450} = 15\sqrt{2}$$

$\therefore$ Required area of a parallelogram

$$= |\vec{a} \times \vec{b}| = 15\sqrt{2} \text{ sq. unit}$$

**12.** Find the area of the triangle with vertices A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5).

➥ A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5) are vertices of $\Delta$ABC.




$\therefore \vec{OA} = \hat{i} + \hat{j} + 2\hat{k}$

$\vec{OB} = 2\hat{i} + 3\hat{j} + 5\hat{k}$

$\vec{OC} = \hat{i} + 5\hat{j} + 5\hat{k}$

$\therefore \vec{AB} = \vec{OB} - \vec{OA}$

$$= 2\hat{i} + 3\hat{j} + 5\hat{k} - (\hat{i} + \hat{j} + 2\hat{k})$$

$$= \hat{i} + 2\hat{j} + 3\hat{k}$$

$\vec{AC} = \vec{OC} - \vec{OB}$

$$= (\hat{i} + 5j + 5\hat{k}) - (\hat{i} + \hat{j} + 2\hat{k})$$

$$= 4\hat{j} + 3\hat{k}$$

$$\text{Now } \vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix}$$

$= \hat{i}(6-12) - \hat{j}(3-0) + \hat{k}(4-0)$

$= -6\hat{i} - 3\hat{j} + 4\hat{k}$

$|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-6)^2 + (-3)^2 + (4)^2}$

$= \sqrt{36+9+16}$

$= \sqrt{61}$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\sqrt{61}$ sq.units

**13.** Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $(2\vec{a} + \vec{b})$ and $(\vec{a} - 3\vec{b})$ externally in the ratio 1 : 2. Also, show that P is the mid point of the line segment RQ.

➥ Given that $\overrightarrow{OP} = 2\vec{a} + \vec{b}$ and $\overrightarrow{OQ} = \vec{a} - 3\vec{b}$

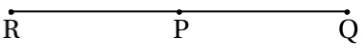
A point R divides the line joining two points P and Q externally in the ration 1 : 2.

$\therefore$ The position vector of R, $\overrightarrow{OR} = \frac{1(\overrightarrow{OQ}) - 2(\overrightarrow{OP})}{1-2}$

$\therefore$ The point P is the midpoint of RQ. $= \frac{\vec{a} - 3\vec{b} - 2(2\vec{a} + \vec{b})}{-1}$

$= \frac{-3\vec{a} - 5\vec{b}}{-1}$

$= 3\vec{a} + 5\vec{b}$




The position vector of the midpoint of RQ.

$= \frac{\overrightarrow{OR} + \overrightarrow{OQ}}{2}$

$= \frac{3\vec{a} + 5\vec{b} + \vec{a} - 3\vec{b}}{2}$

$= \frac{4\vec{a} + 2\vec{b}}{2}$

$= 2\vec{a} + \vec{b} = \overrightarrow{OP}$

= The position vector of P.

$\therefore$ The point P is the midpoint of $\overline{RQ}$.

## Section C

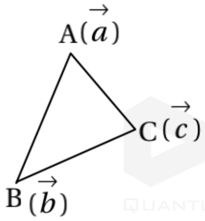
● **Write the answer of the following questions.** [**Each carries 4 Marks**] **[20]**

**14.** Show that the points A, B and C with position vectors, $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$, respectively form the vertices of a right angled triangle.

➥ Here, $\overrightarrow{OA} = \vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$

$\overrightarrow{OB} = \vec{b} = 2\hat{i} - \hat{j} + \hat{k}$

$\overrightarrow{OC} = \vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$




$\therefore \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= (2\hat{i} - \hat{j} + \hat{k}) - (3\hat{i} - 4\hat{j} - 4\hat{k})$

$= -\hat{i} + 3\hat{j} + 5\hat{k}$

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$= (\hat{i} - 3\hat{j} - 5\hat{k}) - (2\hat{i} - \hat{j} + \hat{k})$

$= -\hat{i} - 2\hat{j} - 6\hat{k}$

$\overrightarrow{CA} = \overrightarrow{OA} - \overrightarrow{OC}$

$= (3\hat{i} - 4\hat{j} - 4\hat{k}) - (\hat{i} - 3\hat{j} - 5\hat{k})$

$= 2\hat{i} - \hat{j} + \hat{k}$

$|\overrightarrow{AB}| = \sqrt{(-1)^2 + (3)^2 + (5)^2} = \sqrt{1 + 9 + 25} = \sqrt{35}$

$|\overrightarrow{BC}| = \sqrt{(-1)^2 + (-2)^2 + (-6)^2} = \sqrt{1 + 4 + 36} = \sqrt{41}$

$|\overrightarrow{CA}| = \sqrt{(2)^2 + (-1)^2 + (1)^2} = \sqrt{4 + 1 + 1} = \sqrt{6}$

$\therefore |\overrightarrow{AB}| + |\overrightarrow{CA}|^2 = (\sqrt{35})^2 + (\sqrt{6})^2$

$\therefore AB^2 + CA^2 = 35 + 6$

$\therefore AB^2 + CA^2 = 41$

$= |\overrightarrow{BC}|^2 = BC^2$

$\therefore$ Using the inverse of pythagora's theorem, A, B and C are vertices of a right angled triangle.

**15.** Show that each of the given three vectors is a unit vector :

$\frac{1}{7}\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$, $\frac{1}{7}\left(3\hat{i} - 6\hat{j} + 2\hat{k}\right)$, $\frac{1}{7}\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)$.

Also, show that they are mutually perpendicular to each other.

➥ Let $\vec{a} = \frac{1}{7}(2\hat{i} + 3\hat{j} + 6\hat{k})$

$\vec{b} = \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k})$

$\vec{c} = \frac{1}{7}(6\hat{i} + 2\hat{j} - 3\hat{k})$

$\therefore \vec{a} = \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$

$\vec{b} = \frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k}$

$\vec{c} = \frac{6}{7}\hat{i} + \frac{2}{7}\hat{j} - \frac{3}{7}\hat{k}$

Now $|\vec{a}| = \sqrt{\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2}$

$= \sqrt{\frac{4+9+36}{49}} = 1$

$|\vec{b}| = \sqrt{\left(\frac{3}{7}\right)^2 + \left(\frac{-6}{7}\right)^2 + \left(\frac{2}{7}\right)^2}$

$= \sqrt{\frac{9+36+4}{49}} = 1$

$|\vec{c}| = \sqrt{\left(\frac{6}{7}\right)^2 + \left(\frac{2}{7}\right)^2 + \left(\frac{-3}{7}\right)^2}$

$= \sqrt{\frac{36+4+9}{49}} = 1$

$\therefore$ The magnitude of vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ is 1.

$\therefore$ Each vector is a unit vector.

Now $\vec{a} \cdot \vec{b} = \left(\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right) \cdot \left(\frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k}\right)$

$= \frac{6}{49} - \frac{18}{49} + \frac{12}{49}$

$= \frac{6-18+12}{49} = 0$

$\therefore \vec{a} \perp \vec{b}$

$\vec{b} \cdot \vec{c} = \left(\frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k}\right) \cdot \left(\frac{6}{7}\hat{i} + \frac{2}{7}\hat{j} - \frac{3}{7}\hat{k}\right)$

$= \frac{18}{49} - \frac{12}{49} - \frac{6}{49}$

$= \frac{18-12-6}{49} = 0$

$\therefore \vec{b} \perp \vec{c}$

$\vec{a} \cdot \vec{c} = \left(\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right) \cdot \left(\frac{6}{7}\hat{i} + \frac{2}{7}\hat{j} + \frac{-3}{7}\hat{k}\right)$

$= \frac{12}{49} + \frac{6}{49} - \frac{18}{49}$

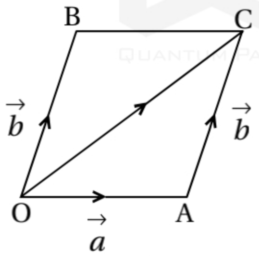
$$= \frac{12 + 6 - 18}{49} = 0$$

$\therefore \vec{a} \perp \vec{c}$

Hence, $\vec{a}, \vec{b}$ and $\vec{c}$ are mutually perpendicular.

**16.** The two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$. Find the unit vector parallel to its diagonal. Also, find its area.

➥ The two adjacent sides of a parallelogram are $\vec{a}$ and $\vec{b}$




$\therefore \overrightarrow{OA} = \vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k}$

$\overrightarrow{OB} = \vec{b} = \hat{i} - 2\hat{j} - 3\hat{k}$

Diagonal $\overrightarrow{OC} = \vec{a} + \vec{b}$

$$= (2\hat{i} - 4\hat{j} + 5\hat{k}) + (\hat{i} - 2\hat{j} - 3\hat{k})$$

$$= 3\hat{i} - 6\hat{j} + 2\hat{k}$$

$\therefore |\overrightarrow{OC}| = \sqrt{(3)^2 + (-6)^2 + (2)^2}$

$= \sqrt{49} = 7$

$\therefore$ The unit vector parallel to the diagonal $\overrightarrow{OC}$

$$= \frac{\overrightarrow{OC}}{|\overrightarrow{OC}|}$$

$$= \frac{3\hat{i} - 6\hat{j} + 2\hat{k}}{7} = \frac{1}{7}(3\hat{i} - 6\hat{j} + 2\hat{k})$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 5 \\ 1 & -2 & -3 \end{vmatrix}$$

$$= 22\hat{j} + 11\hat{j}$$

Area of the parallelogram $= |\vec{a} \times \vec{b}|$

$$= \sqrt{(22)^2 + (11)^2}$$

$$= \sqrt{(11)^2 (2)^2 + (11)^2}$$

$= 11\sqrt{4+1}$

$= 11\sqrt{5}$ sq. units

**17.** If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$.

➥ $\vec{a}, \vec{b}$ and $\vec{c}$ are mutually perpendicular vectors of equal magnitude.

$$\left.\begin{aligned} &\therefore |\vec{a}| = |\vec{b}| = |\vec{c}| = k, (k > 0) \\ &\text{and } \vec{a}\cdot\vec{b} = 0, \vec{b}\cdot\vec{c} = 0, \vec{c}\cdot\vec{a} = 0 \end{aligned}\right\} \text{(i)}$$

$$\text{Now } |\vec{a}+\vec{b}+\vec{c}|^2 = (\vec{a}+\vec{b}+\vec{c})\cdot(\vec{a}+\vec{b}+\vec{c})$$

$$= \vec{a}\cdot\vec{a} + \vec{b}\cdot\vec{b} + \vec{c}\cdot\vec{c} + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{a}\cdot\vec{c})$$

$$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 0$$

$$= k^2 + k^2 + k^2 \quad (\because \text{From (i)})$$

$$= 3k^2$$

$$\therefore |\vec{a}+\vec{b}+\vec{c}| = \sqrt{3}k$$

Let the vector $\vec{a}+\vec{b}+\vec{c}$ makes an angle α, β and γ with the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ respectively.

$$\therefore \cos\alpha = \frac{(\vec{a}+\vec{b}+\vec{c})\cdot\vec{a}}{|\vec{a}+\vec{b}+\vec{c}||\vec{a}|}$$

$$= \frac{\vec{a}\cdot\vec{a} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}}{\sqrt{3}k\cdot k}$$

$$= \frac{|\vec{a}|^2 + 0 + 0}{\sqrt{3}k^2}$$

$$= \frac{k^2}{\sqrt{3}k^2} = \frac{1}{\sqrt{3}} \quad \text{....(ii)}$$

$$\text{Similarly } \cos\beta = \frac{(\vec{a}+\vec{b}+\vec{c})\cdot\vec{b}}{|\vec{a}+\vec{b}+\vec{c}||\vec{b}|}$$

$$= \frac{\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{b} + \vec{c}\cdot\vec{b}}{\sqrt{3}k\cdot k}$$

$$\cos\beta = \frac{0 + |\vec{b}|^2 + 0}{\sqrt{3}k^2}$$

$$= \frac{k^2}{\sqrt{3}k^2} = \frac{1}{\sqrt{3}} \quad \text{...(iii)}$$

$$\text{and } \cos\gamma = \frac{(\vec{a}+\vec{b}+\vec{c})\cdot\vec{c}}{|\vec{a}+\vec{b}+\vec{c}||\vec{c}|}$$

$= \frac{1}{\sqrt{3}}$ ....(iv)

From results (ii), (iii) and (iv),

$\cos\alpha = \cos\beta = \cos\gamma = \frac{1}{\sqrt{3}}$

$\therefore\ \alpha = \beta = \gamma$

$\therefore\ \vec{a} + \vec{b} + \vec{c}$ is equally inclined to

$\vec{a}, \vec{b}$ and $\vec{c}$.

**18.** The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of $\lambda$.

➥ $\vec{a} = 2\hat{i} + 4\hat{j} - 5\hat{k}$

$\vec{b} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$

$\therefore\ \vec{a} + \vec{b} = (\lambda + 2)\hat{i} + 6\hat{j} - 2\hat{k}$

$|\vec{a} + \vec{b}| = \sqrt{(\lambda + 2)^2 + (6)^2 + (-2)^2}$

$= \sqrt{(\lambda + 2)^2 + 40}$

The unit vector in the direction of $\vec{a} + \vec{b}$

$= \frac{\vec{a} + \vec{b}}{|\vec{a} + \vec{b}|}$

$= \frac{(\lambda + 2)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(\lambda + 2)^2 + 40}}$

Now $\vec{c} = \hat{i} + \hat{j} + \hat{k}$

Given that $\vec{c} \cdot \frac{(\vec{a} + \vec{b})}{|\vec{a} + \vec{b}|} = 1$

$\therefore\ \frac{(\hat{i} + \hat{j} + \hat{k}) \cdot ((\lambda + 2)\hat{i} + 6\hat{j} - 2\hat{k})}{\sqrt{(\lambda + 2)^2 + 40}} = 1$

$\therefore\ \lambda + 2 + 6 - 2 = \sqrt{(\lambda + 2)^2 + 40}$

$\therefore\ (\lambda + 6)^2 = (\lambda + 2)^2 + 40$

$\therefore\ \lambda^2 + 12\lambda + 36 = \lambda^2 + 4\lambda + 44$

$\therefore\ 8\lambda = 8$

$\therefore\ \lambda = 1$