#### CHAPTER 9

(A) 2

(A) 3

(A) 0

(A) 3

are .....

1.

2.

3.

8.

# OPEN STUDENT FOUNDATION Std 12 : MATHS

# PRACTICE SHEET DAY 8

Date: 26/02/24

(D) not defined

(D) not defined

(D) 4

(D) 0

[4]

[1]

Section A

The number of arbitary constants in the general solution of a differential equation of fourth order

The number of arbitary constants in the particular solution of a differential equation of third order

**Section B** 

(C) 3

(C) 1

The degree of the differential equation  $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$  is .......

Choose correct answer from the given options. [Each carries 1 Mark]

(B) 1

(B) 2

(B) 2

Write the answer of the following questions. [Each carries 1 Mark]

The order of the differential equation  $2x^2 \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$  is .......

1.	Determine order and degree (if defined) of differential equation : $\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$					
Section C						
•	Write the answer of the following questions. [Each carries 2 Marks]					
2.	For the differential equation, Find the general solution : $(e^x + e^{-x}) dy - (e^x - e^{-x}) dx = 0$					
3.	For the differential equation, Find the general solution : $\frac{dy}{dx} = (1 + x^2)(1 + y^2)$					
4.	Verify that the given function (implicit or explicit) is a solution of the corresponding differential					
	equation: $y = e^x (a \cos x + b \sin x) : \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$					
5.	Verify that the given function (implicit or explicit) is a solution of the corresponding differential					
	equation: $y = ae^x + be^{-x} + x^2$ : $x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} - xy + x^2 - 2 = 0$					
Section D						
•	Write the answer of the following questions. [Each carries 3 Marks]					
6.	For the differential equation, Find the general solution : $e^x \tan y  dx + (1 - e^x) \sec^2 y  dy = 0$					
7.	For differential equation, find the general solution : $\frac{dy}{dx} + \frac{y}{x} = x^2$					

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For differential equation, find the general solution :  $(x + 3y^2) \frac{dy}{dx} = y$ 

9. Solve the differential equation  $\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right] \frac{dx}{dy} = 1 \ (x \neq 0).$ 

#### Section E

• Write the answer of the following questions. [Each carries 4 Marks]

[32]

- 10. show that the given differential equation is homogeneous and solve it:  $x^2 \frac{dy}{dx} = x^2 2y^2 + xy$
- show that the given differential equation is homogeneous and solve it:  $\left\{x\cos\left(\frac{y}{x}\right) + y\sin\left(\frac{y}{x}\right)\right\} y dx$   $= \left\{y\sin\left(\frac{y}{x}\right) x\cos\left(\frac{y}{x}\right)\right\} x dy$
- 12. show that the given differential equation is homogeneous and solve it:  $(x^2 + xy) dy = (x^2 + y^2) dx$
- 13. show that the given differential equation is homogeneous and solve it:  $(1 + e^{\frac{x}{y}}) dx + e^{\frac{x}{y}} \left(1 \frac{x}{y}\right) dy = 0$
- 14. show that the given differential equation is homogeneous and solve it:  $x \frac{dy}{dx} y + x \sin\left(\frac{y}{x}\right) = 0$
- 15. For differential equation, find the general solution :  $x \frac{dy}{dx} + y x + xy \cot x = 0 \ (x \neq 0)$
- Prove that  $x^2 y^2 = c(x^2 + y^2)^2$  is the general solution of differential equation  $(x^3 3xy^2)$   $dx = (y^3 3x^2y)$ dy, where c is a parameter.
- 17. Show that the general solution of the differential equation  $\frac{dy}{dx} + \frac{y^2 + y + 1}{x^2 + x + 1} = 0$  is given by (x + y + 1) = A (1 x y 2xy), where A is parameter.

### CHAPTER 9

# OPEN STUDENT FOUNDATION Std 12: MATHS PRACTICE SHEET DAY 8

Section A

•	Choose correct answer fr	com the given opti	ons. [Each carries 1 Mark]	[4]		
1.	The order of the differential equation $2x^2 \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$ is					
	(A) 2	(B) 1	(C) 0	(D) not defined		
Ans.	(A) 2					
<b>\</b>	The order of the given differential equation is 2.					
2.	The degree of the differ	= 0 is				
	(A) 3	(B) 2	(C) 1	(D) not defined		
Ans.	(D) not defined					
<b>→</b>	The given differential equation is not a polynomial equation in its derivatives so its degree is not defined.					
3.	are		general solution of a different	•		
<b>A</b>	(A) 0	(B) 2	(C) 3	(D) 4		
Ans.						
<b>→</b>	The order of the different of a differential equation	_	so the number of arbitary const	cants in the general solution		
	:. Alternate (D)					
4.	The number of arbitary constants in the particular solution of a differential equation of third order are					
	(A) 3	(B) 2	(C) 1	(D) 0		
Ans.	(D) 0					
<b>→</b>	The particular solution of arbitary constants is 0.	of a differential eq	uation is free from arbitary co	onstants, so the number of		
	:. Alternate (D)					
	Section B					
•	Write the answer of the following questions. [Each carries 1 Mark]					
1.	Determine order and degree (if defined) of differential equation : $\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$					
<b>-</b>	The heighest order derivative present in the given differential equation is $\frac{d^2s}{dt^2}$ . so its degree is 2. The					
	heighest power raised to	$\frac{d^2s}{dt^2}$ is one. so i	its degree is one.			

• Write the answer of the following questions. [Each carries 2 Marks]

- 2. For the differential equation, Find the general solution :  $(e^x + e^{-x}) dy (e^x e^{-x}) dx = 0$
- $\Rightarrow$   $(e^x + e^{-x}) dy (e^x e^{-x}) dx = 0$ 
  - $(e^x + e^{-x}) dy = (e^x e^{-x}) dx$
  - $\therefore dy = \frac{(e^x e^{-x})}{(e^x + e^{-x})} dx$



Integrating both sides, we get

$$\int dy = \int \left( \frac{e^x - e^{-x}}{e^x + e^{-x}} \right) dx$$

$$\therefore \int dy = \int \frac{\frac{d}{dx} (e^x + e^{-x})}{e^x + e^{-x}} dx \quad \left( \text{Use } \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c \right)$$

$$\therefore y = \log |e^x + e^{-x}| + c$$

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which is the general solution of the given differential equation.

3. For the differential equation, Find the general solution :  $\frac{dy}{dx} = (1 + x^2)(1 + y^2)$ 

$$\frac{dy}{dx} = (1+x^2)(1+y^2)$$

$$\therefore \frac{dy}{1+y^2} = (1+x^2) dx$$



Integrating both sides, we get

$$\int \frac{dy}{1+y^2} = \int (1+x^2) \ dx$$

$$\therefore \tan^{-1} y = x + \frac{x^3}{3} + c$$



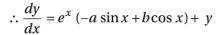
which is the general solution of the given differential equation.

- 4. Verify that the given function (implicit or explicit) is a solution of the corresponding differential equation :  $y = e^x (a \cos x + b \sin x) : \frac{d^2y}{dx^2} 2\frac{dy}{dx} + 2y = 0$
- $\Rightarrow$   $y = e^x (a \cos x + b \sin x)$



Differentiating w.r. to x, we have

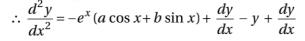
$$\therefore \frac{dy}{dx} = e^x \left( -a \sin x + b \cos x \right) + e^x \left( a \cos x + b \sin x \right)$$





Again Differentiating w.r. to x, we have

$$\frac{d^2y}{dx^2} = e^x \left(-a\cos x - b\sin x\right) + e^x \quad (-a\sin x + b\cos x) + \frac{dy}{dx}$$



$$\therefore \frac{d^2y}{dx^2} = -y + \frac{2dy}{dx} - y$$

(∵ From (1))

$$\therefore \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$



Therefore, the given function is a solution of given differential equation.

Verify that the given function (implicit or explicit) is a solution of the corresponding differential equation :  $y = ae^x + be^{-x} + x^2$ :  $x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} - xy + x^2 - 2 = 0$ 

... (2)

 $\Rightarrow$   $xy = ae^x + be^{-x} + x^2 \dots (1)$ 

Differentiating eq. (1) w.r. to x, we have

$$x\frac{dy}{dx} + y = ae^x - be^{-x} + 2x$$

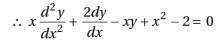
Again differentiating w.r. to x, we get

$$x\frac{dy^2}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx} = ae^x + be^{-x} + 2$$
 ... (3)

From eg. (1) 
$$\Rightarrow ae^x + be^{-x} = xy - x^2$$

This value is substitue in (3), we get

$$x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = xy - x^2 + 2$$



Therefore, the given function is a solution of given differential equation.

## Section D

• Write the answer of the following questions. [Each carries 3 Marks]



- 6. For the differential equation, Find the general solution :  $e^x \tan y \, dx + (1 e^x) \sec^2 y \, dy = 0$
- $\Rightarrow e^x \tan y \ dx + (1 e^x) \ sec^2 y \ dy = 0$

$$\therefore \frac{e^x}{1 - e^x} dx + \frac{\sec^2 y}{\tan y} dy = 0$$

Integrating the equation, we have

$$\int \frac{e^x}{1 - e^x} dx + \int \frac{\sec^2 y}{\tan y} dy = 0$$

$$\therefore -\int \frac{\frac{d}{dx} (1 - e^x)}{1 - e^x} dx + \int \frac{\frac{d}{dy} (\tan y)}{\tan y} dy = 0$$

$$\therefore -\log |1 - e^x| + \log |\tan y| = \log c \left( \text{Use } \int \frac{f'(x)}{f(x)} dx = \log |f(x)| + c \right)$$

$$\therefore \log \left| \frac{\tan y}{1 - e^x} \right| = \log c$$

$$\therefore \frac{\tan y}{1 - e^x} = c$$



:. tan  $y = c(1 - e^x)$ . which is the general solution of the given differential equation.

- 7. For differential equation, find the general solution :  $\frac{dy}{dx} + \frac{y}{x} = x^2$

This is a linear differential equation of the type  $\frac{dy}{dx} + Py = Q$ 

where  $P = \frac{1}{x}$  and  $Q = x^2$ 

$$\therefore \text{ I.F.} = e^{\int P dx} = e^{\int \frac{1}{x} dx}$$
$$= e^{\log_e x} = x$$

Multiplying eq. (1) by x, we have

$$x\frac{dy}{dx} + x \cdot \left(\frac{y}{x}\right) = x \cdot x^2$$

$$\therefore x \frac{dy}{dx} + y = x^3$$

$$\therefore \frac{d}{dx}(xy) = x^3$$

Integrating both sides with respect to x,

$$xy = \int x^3 dx$$

$$\therefore xy = \frac{x^4}{4} + c$$

Which is a general solution of the given differential equation.

8. For differential equation, find the general solution :  $(x + 3y^2) \frac{dy}{dx} = y$  (y > 0)

$$(x + 3y^2) \frac{dy}{dx} = y$$

$$\therefore \frac{(x+3y^2)}{y} = \frac{dx}{dy}$$

$$\therefore \frac{dx}{dy} - \frac{x}{y} = 3y \qquad \dots (1)$$

.. This is a linear equation of the

$$\frac{dx}{dy} + P_1 x = Q_1.$$

where  $P_1 = -\frac{1}{y}$  and  $Q_1 = 3y$ 

I.F. = 
$$e^{\int P_1 dy}$$
  
=  $e^{\int -\frac{1}{y} dy}$   
=  $e^{-\log y} = \frac{1}{y}$ 

Multiplying eq. (1) by  $\frac{1}{y}$ , we get

$$\frac{1}{y}\frac{dx}{dy} - \frac{1}{y^2}x = \frac{1}{y}(3y)$$

$$\therefore \frac{d}{dy} \left( \frac{1}{y} \cdot x \right) = 3$$

Integrating w.r. to y, we have

$$\frac{x}{y} = \int 3dy$$

$$\therefore \frac{x}{y} = 3y + c$$

$$\therefore x = 3y^2 + cy$$

Which is the general solution of the given differential equation.



9. Solve the differential equation  $\left[\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right] \frac{dx}{dy} = 1 \ (x \neq 0).$ 

$$\therefore \frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}} = \frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} + \frac{1}{\sqrt{x}} y = \frac{e^{-2\sqrt{x}}}{\sqrt{x}} \qquad \dots (1)$$

This is a linear equation of the type  $\frac{dy}{dx} + Py = Q$ 

Where 
$$P = +\frac{1}{\sqrt{x}}$$
,  $Q = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$ 

Integrating Factor (I.F.) =  $e^{\int P dx}$ 

$$=e^{\int +\frac{1}{\sqrt{x}} dx}$$

$$=e^{2\sqrt{x}}$$

Multiplying eq. (1) by  $e^{2\sqrt{x}}$ , we get

$$\frac{dy}{dx} e^{2\sqrt{x}} + \frac{1}{\sqrt{x}} \cdot e^{2\sqrt{x}} \cdot y = \frac{e^{-2\sqrt{x}}}{\sqrt{x}} \times e^{2\sqrt{x}}$$

$$\therefore \frac{d}{dx} \left( y \cdot e^{2\sqrt{x}} \right) = \frac{1}{\sqrt{x}}$$

Integrating, we get

$$y \cdot e^{2\sqrt{x}} = \int \frac{1}{\sqrt{x}} \, dx$$

$$\therefore y \cdot e^{2\sqrt{x}} = 2\sqrt{x} + c$$

Which is the solution of the given differential equation.

#### Section E

• Write the answer of the following questions. [Each carries 4 Marks]

[32]

- 10. show that the given differential equation is homogeneous and solve it:  $x^2 \frac{dy}{dx} = x^2 2y^2 + xy$
- $\Rightarrow x^2 \frac{dy}{dx} = x^2 2y^2 + xy$

$$\therefore \frac{dy}{dx} = \frac{x^2 - 2y^2 + xy}{x^2}$$

... (1

Let 
$$F(x, y) = \frac{x^2 - 2y^2 + xy}{x^2}$$
 :  $F(\lambda x, \lambda y) = \frac{\lambda^2 x^2 - 2\lambda^2 y^2 + \lambda^2 xy}{\lambda^2 x^2}$ 

$$= \frac{\lambda^2 (x^2 - 2y^2 + xy)}{\lambda^2 x^2} = \lambda^0 F(x, y)$$

Thus, F(x, y) is a homogeneous function of degree zero. Therefore, the given differential equation is a homogeneous differential equation.

Take 
$$y = vx$$
,  $\frac{dy}{dx} = v + x \frac{dv}{dx}$ 

Substitute this value in eq. (1), we get

$$v + x \frac{dv}{dx} = \frac{x^2 - 2v^2x^2 + vx^2}{x^2}$$

$$\therefore x \frac{dv}{dx} = 1 - 2v^2 + v - v$$

$$\therefore x \frac{dv}{dx} = 1 - 2v^2 = -(2v^2 - 1)$$

$$\therefore \frac{dv}{2v^2 - 1} = -\frac{dx}{x}$$

Integrating both sides, we get

$$\int \frac{dv}{2v^2 - 1} = \int -\frac{dx}{x}$$

$$\therefore \frac{1}{2} \int \frac{dv}{v^2 - \left(\frac{1}{\sqrt{2}}\right)^2} = -\int \frac{dx}{x}$$

$$\therefore \frac{1}{2} \times \frac{1}{2\left(\frac{1}{\sqrt{2}}\right)} \log \left| \frac{v - \frac{1}{\sqrt{2}}}{v + \frac{1}{\sqrt{2}}} \right| = -\log x + c$$

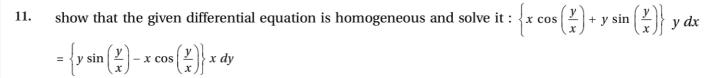
Now replace  $v = \frac{y}{x}$ ,

$$\therefore \frac{1}{2\sqrt{2}} \log \left| \frac{\frac{y}{x} - \frac{1}{\sqrt{2}}}{\frac{y}{x} + \frac{1}{\sqrt{2}}} \right| = -\log x + c$$

$$\therefore \frac{1}{2\sqrt{2}} \log \left| \frac{\sqrt{2}y - x}{\sqrt{2}y + x} \right| = -\log x + c$$

$$\therefore \frac{1}{2\sqrt{2}} \log \left| \frac{x + \sqrt{2}y}{x - \sqrt{2}y} \right| = -\log x + c$$

Which is a required general solution.



$$\Rightarrow \left\{ x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right) \right\} y \ dx = \left\{ y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right) \right\} x \ dy$$

$$\therefore \frac{dy}{dx} = \frac{y \left[ x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right) \right]}{x \left[ y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right) \right]} \qquad \dots (1)$$

Let F(x, y) = 
$$\frac{y \left[ x \cos\left(\frac{y}{x}\right) + y \sin\left(\frac{y}{x}\right) \right]}{x \left[ y \sin\left(\frac{y}{x}\right) - x \cos\left(\frac{y}{x}\right) \right]}$$

$$\therefore F(\lambda x, \lambda y) = \frac{\lambda y \left[ \lambda x \cos\left(\frac{y}{x}\right) + \lambda y \sin\left(\frac{y}{x}\right) \right]}{\lambda x \left[ \lambda y \sin\left(\frac{y}{x}\right) - \lambda x \cos\left(\frac{y}{x}\right) \right]}$$

$$= \frac{\lambda^2 y \left[ x \cos \left( \frac{y}{x} \right) + y \sin \left( \frac{y}{x} \right) \right]}{\lambda^2 x \left[ y \sin \left( \frac{y}{x} \right) - x \cos \left( \frac{y}{x} \right) \right]}$$
$$= \lambda^0 \text{ F } (x, y)$$

Thus, F(x, y) is a homogeneous function of degree zero. Therefore, the given differential equation is a homogeneous differential equation.

$$y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substitute this value in eq. (1), we have

$$v + x \frac{dv}{dx} = \frac{vx \left[ x \cos v + vx \sin v \right]}{x \left[ vx \sin v - x \cos v \right]}$$

$$\therefore x \frac{dv}{dx} = \frac{v[\cos v + v \sin v]}{v \sin v - \cos v} - v$$

$$\therefore x \frac{dv}{dx} = \frac{2v \cos v}{v \sin v - \cos v}$$

$$\therefore \left( \frac{v \sin v - \cos v}{2v \cos v} \right) dv = \frac{dx}{x}$$

Integrating both sides, we get



$$\int \left( \frac{v \sin v - \cos v}{2v \cos v} \right) dv = \int \frac{dx}{x}$$

$$\therefore \frac{1}{2} \int \tan v \, dv - \frac{1}{2} \int \frac{1}{v} \, dv = \int \frac{dx}{x}$$

$$\therefore \frac{1}{2} \log |\sec v| - \frac{1}{2} \log |v| = \log |x| + \log c_1$$

Now replace  $v = \frac{y}{x}$ ,

$$\frac{1}{2} \log \left| \sec \frac{y}{x} \right| - \frac{1}{2} \log \left| \frac{y}{x} \right| = \log |x| + \log c_1$$

$$\therefore -\frac{1}{2} \log \left| \cos \frac{y}{x} \right| - \frac{1}{2} \log \left| \frac{y}{x} \right| - \log |x| = \log c_1$$

$$\therefore \log \left| \cos \frac{y}{x} \right| + \log \left| \frac{y}{x} \right| + 2 \log x = -2 \log c_1$$

$$\therefore \cos\left(\frac{y}{x}\right) \times \frac{y}{x} \times x^2 = c$$

$$\therefore xy \cos\left(\frac{y}{x}\right) = c$$

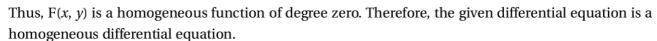
12. show that the given differential equation is homogeneous and solve it:  $(x^2 + xy) dy = (x^2 + y^2) dx$ 

$$(x^2 + xy) dy = (x^2 + y^2) dx$$

$$\therefore \frac{dy}{dx} = \frac{x^2 + y^2}{x^2 + xy}$$

Let 
$$F(x, y) = \frac{x^2 + y^2}{x^2 + xy}$$

$$\therefore F(\lambda x, \lambda y) = \frac{\lambda^2 x^2 + \lambda^2 y^2}{\lambda^2 x^2 + \lambda^2 xy}$$
$$= \frac{\lambda^2 (x^2 + y^2)}{\lambda^2 (x^2 + xy)} = \lambda^0 F(x, y)$$



Take y = vx,

$$\Rightarrow \frac{dy}{dx} = v + x \frac{dy}{dx}$$

Substitute this value in eq. (1), we get

$$v + x \frac{dv}{dx} = \frac{x^2 + v^2 x^2}{x^2 + v x^2}$$

$$\therefore x \frac{dv}{dx} = \frac{1+v^2}{1+v} - v$$

$$\therefore x \frac{dv}{dx} = \frac{1-v}{1+v}$$

$$\therefore x \frac{dv}{dx} = -\left(\frac{v-1}{v+1}\right)$$

$$\therefore \left(\frac{v+1}{v-1}\right) dv = -\frac{dx}{x}$$

Integrating both sides, we get

$$\int \left(\frac{v+1}{v-1}\right) dv = -\int \frac{dx}{x}$$

$$\therefore \int \left(\frac{v-1+2}{v-1}\right) dv = -\int \frac{dx}{x}$$

$$\therefore \int \left(1 + \frac{2}{v-1}\right) dv = -\int \frac{dx}{x}$$

$$\therefore v + 2 \log (v - 1) = -\log x + c_1$$

Replace  $v = \frac{y}{x}$ , we get

$$\frac{y}{x} + 2\log\left(\frac{y}{x} - 1\right) = -\log x + c_1$$

$$\therefore \frac{y}{x} + 2\log(y - x) - 2\log x + \log x = c_1$$

$$\therefore \frac{y}{x} + 2 \log(y - x) - \log x = c_1$$

$$\therefore 2 \log (y - x) - \log x = c_1 - \frac{y}{x}$$

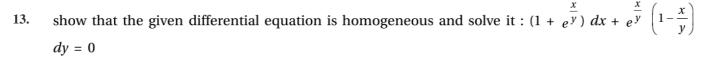
$$\therefore \log \left[ \frac{(y-x)^2}{x} \right] = c_1 - \frac{y}{x}$$

$$\therefore \frac{(y-x)^2}{x} = e^{c_1 - \frac{y}{x}} = e^{c_1} \cdot e^{-\frac{y}{x}} = c e^{-\frac{y}{x}}$$

$$\therefore (y-x)^2 = x \cdot c \cdot e^{-\frac{y}{x}}$$

$$\therefore (x-y)^2 = c x e^{-\frac{y}{x}} \text{ where, } c = e^c_1$$

Which is the general solution of the given differential equation.



$$\Rightarrow (1 + e^{\frac{x}{y}}) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0$$

$$\therefore \frac{dx}{dy} = \frac{-e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right)}{1 + e^{\frac{x}{y}}}$$

$$F(x, y) = \frac{e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right)}{\frac{x}{y}}$$
 is a homogeneous function.







$$1 + e^y$$

:. The given differential equation is a homogeneous differential equation.

Take 
$$\frac{x}{y} = v \Rightarrow x = vy$$

Differentiate w.r. to y, we have

$$\frac{dx}{dy} = v + y \, \frac{dv}{dy}$$

Substitute this value in eq. (1), we get

$$v + y \frac{dv}{dy} = \frac{-e^{v}(1-v)}{1+e^{v}}$$

$$\therefore y \frac{dv}{dy} = \frac{-e^v + v e^v}{1 + e^v} - v$$

$$\therefore y \frac{dv}{dy} = \frac{-e^{v} - v}{1 + e^{v}} = -\frac{(e^{v} + v)}{1 + e^{v}}$$

$$\therefore \left(\frac{1+e^{v}}{e^{v}+v}\right) dv = -\frac{dy}{y}$$

Integrating both sides, we get

$$\int \left(\frac{1+e^{\nu}}{\nu+e^{\nu}}\right) d\nu = \int \frac{-dy}{y}$$

$$\therefore \log (v + e^{v}) = -\log y + \log c \left(\because \frac{d}{dv} (v + e^{v}) = 1 + e^{v}\right)$$

$$\therefore \log (v + e^{v}) + \log y = \log c$$

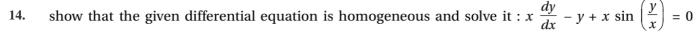
$$\therefore y (v + e^v) = c$$

Now replace  $v = \frac{x}{v}$ ,

$$y\left(\frac{x}{y} + e^{\frac{x}{y}}\right) = c$$

$$\therefore x + y e^{\frac{x}{y}} = c$$

Which is a required general solution.



$$\Rightarrow x \frac{dy}{dx} - y + x \sin\left(\frac{y}{x}\right) = 0$$

$$\therefore \frac{dy}{dx} = \frac{y - x \sin\left(\frac{y}{x}\right)}{x} \qquad \dots (1)$$

Let 
$$F(x, y) = \frac{y - x \sin\left(\frac{y}{x}\right)}{x}$$











$$\therefore F(\lambda x, \lambda y) = \frac{\lambda y - \lambda x \sin\left(\frac{y}{x}\right)}{\lambda x} = \lambda^0 F(x, y)$$



Thus, F(x, y) is a homogeneous function of degree zero. Therefore, the given differential equation is a homogeneous differential equation.

Take 
$$y = vx$$
,  $\frac{dy}{dx} = v + x \frac{dv}{dx}$ 

Substitute this value in eq. (1), we get

$$v + x \frac{dv}{dx} = \frac{vx - x \sin v}{x}$$

$$\therefore x \frac{dv}{dx} = v - \sin v - v$$

$$\therefore \frac{dv}{\sin v} = -\frac{dx}{x}$$

Integrating both sides, we get

$$\int \csc v \, dv = -\int \frac{dx}{x}$$



$$\therefore \log |\csc v - \cot v| = -\log x + \log c$$

$$\therefore \csc v - \cot v = \frac{c}{x}$$

Now replace 
$$v = \frac{y}{x}$$

$$\csc \frac{y}{x} - \cot \frac{y}{x} = \frac{c}{x}$$

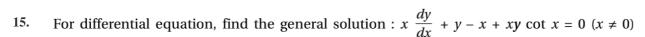


$$\therefore \frac{1 - \cos\left(\frac{y}{x}\right)}{\sin\left(\frac{y}{x}\right)} = \frac{c}{x}$$

$$\therefore x \left[ 1 - \cos \left( \frac{y}{x} \right) \right] = c \sin \left( \frac{y}{x} \right)$$



Which is a required general solution.



$$\Rightarrow x \frac{dy}{dx} + y - x + xy \cot x = 0$$

$$\therefore x \frac{dy}{dx} + y (1 + x \cot x) = x$$

$$\therefore \frac{dy}{dx} + y \left( \frac{1 + x \cot x}{x} \right) = 1$$

This is a linear equation of the form  $\frac{dy}{dx} + Py = Q$ .

where 
$$P = \frac{1 + x \cot x}{x}$$
 and  $Q = 1$ 

I.F. = 
$$e^{\int P dx}$$
  
=  $e^{\int \left(\frac{1}{x} + \cot x\right) dx}$ 



$$= e^{\log x + \log \sin x}$$

$$=e^{\log(x \sin x)}$$

$$= x \sin x$$

Multiplying eq. (1) by  $x \sin x$ ,

$$x \sin x \frac{dy}{dx} + x \sin x \cdot y \left( \frac{1 + x \cot x}{x} \right) = x \sin x$$

$$\therefore x \sin x \frac{dy}{dx} + (\sin x + x \cos x) = x \sin x$$

$$\therefore \frac{d}{dx} ((x \sin x) \cdot y) = x \sin x$$

Integrating w.r. to x, we get

$$\therefore (x \sin x) \cdot y = \int x \sin x \cdot dx$$

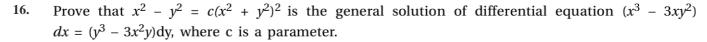
$$\therefore (x \sin x) y = x \int \sin x \cdot dx - \int \left(\frac{d}{dx}(x) \int \sin x \, dx\right) dx$$

$$\therefore (x \sin x) \ y = -x \cos x - \int -\cos x \ dx$$

$$\therefore (x \sin x) y = -x \cos x + \sin x + c$$

$$\therefore y = -\cot x + \frac{1}{x} + \frac{c}{x \sin x}$$

Which is the general solution of the given differential equation.



... (1)

$$\Rightarrow$$
  $(x^3 - 3xy^2) dx = (y^3 - 3x^2y) dy$ 

$$\therefore \frac{dy}{dx} = \frac{x^3 - 3xy^2}{y^3 - 3x^2y}$$

It is a homogeneous differential equation.

Take 
$$y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

Substitute this value in eq. (1), we have

$$v + x \frac{dv}{dx} = \frac{x^3 - 3x^3 v^2}{x^3 v^3 - 3x^3 v}$$

$$\therefore x \frac{dv}{dx} = \frac{1 - 3v^2}{v^3 - 3v} - v$$

$$\therefore x \frac{dv}{dx} = \frac{1 - 3v^2 - v^4 + 3v^2}{v^3 - 3v}$$

$$\therefore x \frac{dv}{dx} = \frac{1 - v^4}{v^3 - 3v}$$

$$\therefore x \frac{dv}{dx} = -\left(\frac{v^4 - 1}{v^3 - 3v}\right)$$

$$\therefore \left(\frac{v^3 - 3v}{v^4 - 1}\right) dv = -\frac{dx}{x}$$















Integrating both sides, we have

$$\int \left(\frac{v^3 - 3v}{v^4 - 1}\right) dv = -\int \frac{dx}{x}$$

$$\therefore \int \frac{v^3}{v^4 - 1} dv - 3 \int \frac{v}{v^4 - 1} dv = -\int \frac{dx}{x}$$

$$\therefore \frac{1}{4} \int \frac{4v^3}{v^4 - 1} - \frac{3}{2} \int \frac{2v}{(v^2)^2 - 1} dv = -\int \frac{dx}{x}$$

$$\therefore \frac{1}{4} \log |v^4 - 1| - \frac{3}{2} \times \frac{1}{2} \log \left| \frac{v^2 - 1}{v^2 + 1} \right| = -\log x + \log k$$

Now replace  $v = \frac{y}{x}$ ,

$$\frac{1}{4} \log \left| \frac{y^4}{x^4} - 1 \right| - \frac{3}{4} \log \left| \frac{\frac{y^2}{x^2} - 1}{\frac{y^2}{x^2} + 1} \right| = -\log x + \log k$$

$$\therefore \frac{1}{4} \log \left| \frac{y^4 - x^4}{x^4} \right| - \frac{3}{4} \log \left| \frac{y^2 - x^2}{y^2 + x^2} \right| = \log \frac{k}{x}$$

$$\therefore \log \left[ \frac{y^4 - x^4}{x^4} \right]^{\frac{1}{4}} - \log \left[ \frac{y^2 - x^2}{y^2 + x^2} \right]^{\frac{3}{4}} = \log \frac{k}{x}$$

$$\left( \frac{y^4 - x^4}{x^4} \right)^{\frac{1}{4}} \times \left( \frac{y^2 + x^2}{y^2 - x^2} \right)^{\frac{3}{4}} = \frac{k}{x}$$

$$\therefore \frac{(y^2 - x^2)^{\frac{1}{4}} (y^2 + x^2)^{\frac{1}{4}}}{x} \times \frac{(y^2 + x^2)^{\frac{3}{4}}}{(y^2 - x^2)^{\frac{3}{4}}} = \frac{k}{x}$$

$$\therefore \frac{(y^2 + x^2)}{(y^2 - x^2)^{\frac{1}{2}}} = k$$

$$\therefore (y^2 + x^2) = k(y^2 - x^2)^{\frac{1}{2}}$$

Squaring both sides, we get

$$(y^2 + x^2)^2 = k^2 (y^2 - x^2)$$

$$\therefore (y^2 - x^2) = \frac{(y^2 + x^2)^2}{k^2}$$

$$\therefore x^2 - y^2 = -\frac{1}{k^2}(x^2 + y^2)^2$$

$$\therefore x^2 - y^2 = c (x^2 + y^2)^2 \text{ where } c = -\frac{1}{k^2}$$

17. Show that the general solution of the differential equation 
$$\frac{dy}{dx} + \frac{y^2 + y + 1}{x^2 + x + 1} = 0$$
 is given by  $(x + y + 1) = A(1 - x - y - 2xy)$ , where A is parameter.

$$\frac{dy}{dx} + \frac{y^2 + y + 1}{x^2 + x + 1} = 0$$

$$\therefore \frac{dy}{y^2+y+1} + \frac{dx}{x^2+x+1}$$

Integrating, we get

$$\int \frac{dy}{y^2 + y + 1} + \int \frac{dx}{x^2 + x + 1} = 0$$

$$\Rightarrow \int \frac{dy}{y^2 + y + \frac{1}{4} + \frac{3}{4}} + \int \frac{dx}{x^2 + x + \frac{1}{4} + \frac{3}{4}} = 0$$

$$\Rightarrow \int \frac{dy}{\left(y + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} + \int \frac{dx}{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 0$$

$$\Rightarrow \frac{2}{\sqrt{3}} \tan^{-1} \left( \frac{y + \frac{1}{2}}{\frac{\sqrt{3}}{2}} \right) + \frac{2}{\sqrt{3}} \tan^{-1} \left( \frac{x + \frac{1}{2}}{\frac{\sqrt{3}}{2}} \right) = c$$

$$\Rightarrow \tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right) + \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) = \frac{\sqrt{3}}{2} c$$

$$\Rightarrow \tan^{-1} \left[ \frac{\frac{2y+1}{\sqrt{3}} + \frac{2x+1}{\sqrt{3}}}{1 - \left(\frac{2y+1}{\sqrt{3}}\right) \left(\frac{2x+1}{\sqrt{3}}\right)} \right] = \frac{\sqrt{3}}{2} c$$

$$\Rightarrow \frac{2y+1+2x+1}{3-(2y+1)(2x+1)} \times \frac{3}{\sqrt{3}} = \tan\left(\frac{\sqrt{3}}{2}c\right)$$

$$\Rightarrow \frac{\sqrt{3} (2x + 2y + 2)}{3 - 4xy - 2x - 2y - 1} = \tan \left(\frac{\sqrt{3}}{2} c\right)$$

$$\Rightarrow \frac{2\sqrt{3}(x+y+1)}{2(1-x-y-2xy)} = \tan\left(\frac{\sqrt{3}}{2}c\right)$$

$$\Rightarrow x + y + 1 = \frac{1}{\sqrt{3}} \tan \left( \frac{\sqrt{3}}{2} c \right) (1 - x - y - 2xy)$$

$$\Rightarrow (x + y + 1) = A(1 - x - y - 2xy)$$

Where 
$$A = \frac{1}{\sqrt{3}} \tan \left( \frac{\sqrt{3}}{2} c \right)$$
 is a parameter.

Which is the required general solution.











