OPEN STUDENT FOUNDATION
Std 12 : MATHS
PRACTICE SHEET DAY 7

Section A

- **Write the answer of the following questions. [Each carries 3 Marks]** **[18]**

1. Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

2. Find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

3. Find the area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$.

4. Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

5. Sketch the graph of $y = |x + 3|$ and evaluate $\int_{-6}^{0} |x+3|\, dx$.

6. Find the area under the given curves and given lines :

(i) $y = x^2$; $x = 1$, $x = 2$ and X-axis

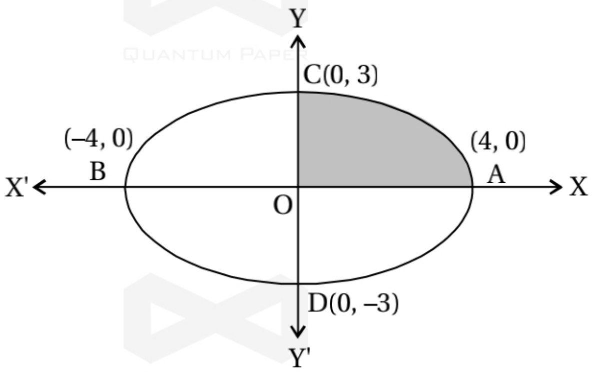
(ii) $y = x^4$; $x = 1$, $x = 5$ and X-axis

Section A

- **Write the answer of the following questions. [Each carries 3 Marks]** **[18]**

1. Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

➥ Ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ is given



$a^2 = 16$, $b^2 = 9 \Rightarrow a > 0$ centre of the ellipse is O(0, 0), $a = 4$ and $b = 3$. So the length of major axis is $2a = 8$ and the length of minor axis is $2b = 6$.

Ellipse is symmetrical about both the axes.

$\therefore$ The area of the ellipse is four times the area of the ellipse is first quadrant.

Now $\frac{x^2}{16} + \frac{y^2}{9} = 1$

$\therefore \frac{y^2}{9} = 1 - \frac{x^2}{16}$

$\therefore y^2 = \frac{9}{16}(16 - x^2)$

$\therefore y = \pm \frac{3}{4}\sqrt{16 - x^2}$ (i)

We find the area of the ellipse in first quadrant.

$\therefore$ The area of the region OACO in first quadrant is A then A = | I |

where $I = \int_0^4 y \, dx$

$= \int_0^4 \frac{3}{4}\sqrt{16 - x^2} \, dx$ (In first quadrant, y is positive)

$= \frac{3}{4}\int_0^4 \sqrt{(4)^2 - x^2} \, dx$

$= \frac{3}{4}\left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$

$$\left\{\because \int\sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c \text{ (Using formula)}\right\}$$

$$= \frac{3}{4}\left[\left(0 + \frac{16}{2}\sin^{-1}1\right) - \left(0 + \frac{16}{2}\sin^{-1}0\right)\right]$$

$$= \frac{3}{4} \times 8 \times \frac{\pi}{2}$$

$$= 3\pi$$

$\therefore$ A = |I| = 3π

$\therefore$ The area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ is Area = 4A = 4(3π) = 12π Sq. units.

2. Find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

➥ Ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is given

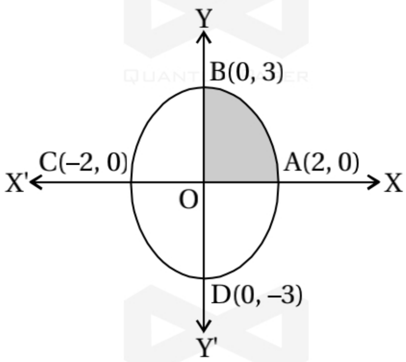
$\therefore a^2 = 4,\ b^2 = 9 \Rightarrow a < b$

$a = 2,\ b = 3$

Centre of the ellipse is O(0, 0). The length of major axis = $2b = 6$. The length of minor of axis = $2a = 4$.

Ellipse is symmetrical about both the axis.

$\therefore$ The area of the ellipse is four times the area of the ellipse in first quadrant



$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

$$\therefore \frac{y^2}{9} = 1 - \frac{x^2}{4}$$

$$\therefore y^2 = \frac{9}{4}(4 - x^2)$$

$$\therefore y = \pm\frac{3}{2}\sqrt{4 - x^2} \qquad \text{....(i)}$$

We find the area of the ellipse in first quadrant.

$\therefore$ The area of the region OABO in first quadrant is A then A = |I|

where $I = \int_0^2 y\, dx$

$= \int_0^2 \frac{3}{2}\sqrt{4 - x^2}\ dx$ ($\because$ In first quadrant y is positive)

$$I = \frac{3}{2}\int_0^2 \sqrt{(2)^2 - x^2}\ dx$$

$$= \frac{3}{2}\left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$$

$$\left(\because \int\sqrt{a^2 - x^2}\ dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c \text{ (Using the formula)}\right)$$

$$= \frac{3}{2}\left[(0 + 2\sin^{-1}1) - (0 + 2\sin^{-1}0)\right]$$

$$= \frac{3}{2} \times 2 \times \frac{\pi}{2} = \frac{3\pi}{2}$$

$$\therefore A = |I| = \frac{3\pi}{2}$$

$\therefore$ The area of region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is $= 4A$

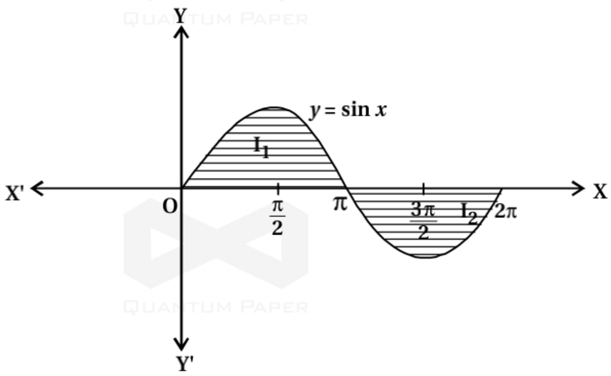
$$= 4\left(\frac{3\pi}{2}\right)$$

$$= 6\pi \text{ Sq. units}$$

3. Find the area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$.

➥ $y = \sin x,\ x = 0$ and $x = 2\pi$

The area bounded by $y = \sin x$, $x = 0$ and $x = 2\pi$ is represented by shaded region in the figure.



$$I_1 = \int_0^{\pi} \sin x\ dx = [-\cos x]_0^{\pi}$$

$$= -[(-1) - (1)] = 2$$

$$I_2 = \int_{\pi}^{2\pi} \sin x\ dx = -[\cos x]_{\pi}^{2\pi}$$

$$= -[1 - (-1)] = -2$$

$\therefore$ Required area $A = |I_1| + |I_2|$

$$= |2| + |-2|$$

$$= 2 + 2$$

$= 4$ Sq. units.

Note : Here in the area I_2, $f(x) < 0$ so the numerical value of I_2 is negative. But area is always positive. So, we take $|I_2|$.

4. Find the area enclosed by the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

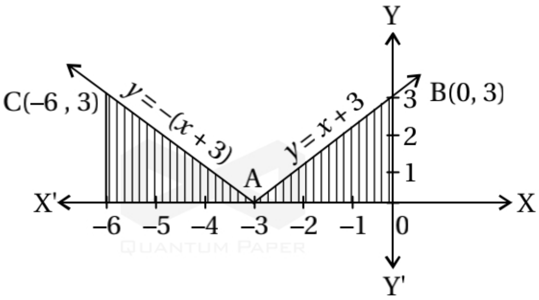
➡ πab

5. Sketch the graph of $y = |x + 3|$ and evaluate $\displaystyle\int_{-6}^{0} |x+3| \, dx$.

➡ $y = |x + 3|$

$= x + 3, \; x \geq -3$

$= -(x + 3), \; x < -3$



Required area is represented by shaded region in the figure.

$$I_1 = \int_{-3}^{0} (x + 3)\, dx = \left[\frac{(x+3)^2}{2}\right]_{-3}^{0} = \frac{1}{2}(9) = \frac{9}{2}$$

$$I_2 = \int_{-6}^{-3} -(x + 3)\, dx = \left[-\frac{(x+3)^2}{2}\right]_{-6}^{-3}$$

$$= -\frac{1}{2}[0 - 9] = \frac{9}{2}$$

$$\therefore \int_{-6}^{-3} |x+3|\, dx = \int_{-6}^{-3} -(x + 3)\, dx + \int_{-3}^{0} (x + 3)\, dx$$

$$= I_2 + I_1$$

$$= \frac{9}{2} + \frac{9}{2}$$

$= 9$ Sq. units

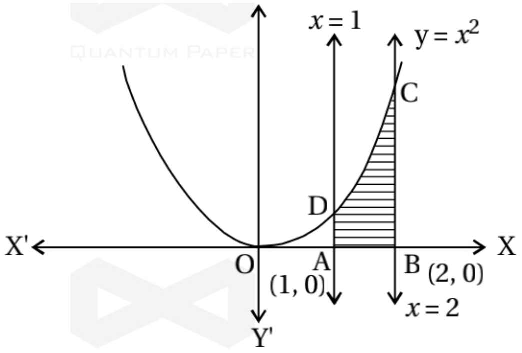
6. Find the area under the given curves and given lines :

(i) $y = x^2$; $x = 1$, $x = 2$ and X-axis

(ii) $y = x^4$; $x = 1$, $x = 5$ and X-axis

➡ **(i) $y = x^2$; $x = 1$, $x = 2$ and X-axis**

Y



$y = x^2$ is a parabola symmetrical about Y-axis.

The area of the region bounded by the curve $y = x^2$, $x = 1$, $x = 2$ and X-axis is ABCDA.

$$\therefore \text{ Required area A } = \int_1^2 x^2 \, dx$$

$$= \left[\frac{x^3}{3}\right]_1^2$$

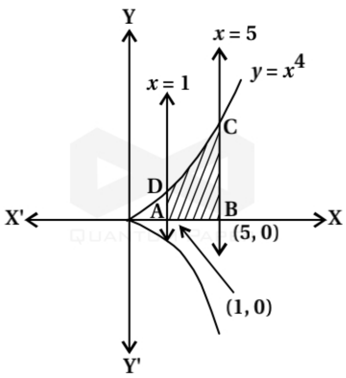
$$= \frac{1}{3}(8 - 1)$$

$$= \frac{7}{3} \text{ Sq. units}$$

(ii) $y = x^4$; $x = 1$, $x = 5$ and X-axis.

➡ $y = x^4$ is a curve symmetrical about X-axis.

The area of the region bounded by the curve $y = x^4$, $x = 1$ and $x = 5$ is ABCDA

➡ 

$$\therefore \text{ Required area A } = \int_1^5 x^4 \, dx$$

$$= \left[\frac{x^5}{5}\right]_1^5$$

$$= \frac{1}{5}(3125 - 1)$$

$$= \frac{3124}{5} \text{ Sq. units}$$

$$= 624.8 \text{ Sq. units}$$