CHAPTER 7

OPEN STUDENT FOUNDATION
Std 12 : MATHS
PRACTICE SHEET DAY 6

Date : 24/02/24

Section A

- **Write the answer of the following questions. [Each carries 2 Marks]** **[22]**

1. Find the following integral : $\int x^2\left(1-\frac{1}{x^2}\right)dx$

2. Find the following integral : $\int \frac{x^3 - x^2 + x - 1}{x - 1}\,dx$

3. Find the following integral : $\int \frac{2 - 3\sin x}{\cos^2 x}\,dx$

4. Integrate the function : $\frac{e^{tan^{-1}x}}{1 + x^2}$

5. Integrate the function : $\frac{x^3\sin(\tan^{-1}x^4)}{1 + x^8}$

6. Integrate the function : $\frac{(1 + \log x)^2}{x}$

7. Find the integral of the function : $\frac{\sin^3 x + \cos^3 x}{\sin^2 x\cos^2 x}$

8. Integrate the function : $\frac{4x + 1}{\sqrt{2x^2 + x - 3}}$

9. Integrate the rational function : $\frac{1}{(e^x - 1)}$ (Note : Take $e^x = t$)

10. Evaluate the definite integral : $\int_0^1\left(x\,e^x + \sin\frac{\pi x}{4}\right)dx$

11. By using the property of definite integral evaluate the integral in : $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^7 x\,dx$

Section B

- **Write the answer of the following questions. [Each carries 3 Marks]** **[66]**

12. Integrate the function : $\frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}$

13. Integrate the function : $\frac{(x + 1)(x + \log x)^2}{x}$

14. Find the integral of the function : $\frac{\cos 2x}{(\cos x + \sin x)^2}$

15. Integrate the function : $\frac{1}{\sqrt{(2 - \mathrm{x})^2 + 1}}$

16. Integrate the rational function : $\frac{1-x^2}{x(1-2x)}$

17. Integrate the rational function : $\frac{3x-1}{(x-1)(x-2)(x-3)}$

18. Integrate the rational function : $\frac{1}{x(x^4-1)}$

19. Integrate the functions : $\sqrt{1+\frac{x^2}{9}}$

20. Evaluate the integral using substitution : $\int_1^2\left(\frac{1}{x}-\frac{1}{2x^2}\right)e^{2x}\,dx$

21. By using the property of definite integral evaluate the integral in : $\int_0^{\pi}\frac{x\,dx}{1+\sin x}$

22. By using the property of definite integral evaluate the integral in : $\int_0^{a}\frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\,dx$

23. Integrate the function : $f'(ax+b)\,[f(ax+b)]^n$

24. Integrate the function : $\frac{1}{(x^2+1)(x^2+4)}$

25. Integrate the function : $\frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}$

26. Integrate the function : $\frac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{2\log x}}$

27. Integrate the function : $\frac{5x}{(x+1)(x^2+9)}$

28. Integrate the function : $\frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}$ $\left[\text{Hint : }\frac{1}{x^{\frac{1}{2}}+x^{\frac{1}{3}}}=\frac{1}{x^{\frac{1}{3}}\left(1+x^{\frac{1}{6}}\right)},\text{ Put } x=t^6\right]$

29. Evaluate the definite integral : $\int_0^{\frac{\pi}{2}}\frac{\cos^2 x\,dx}{\cos^2 x+4\sin^2 x}$

30. Integrate the function : $\frac{\sqrt{x^2+1}\left[\log(x^2+1)-2\log x\right]}{x^4}$

31. Evaluate the definite integral : $\int_{\frac{\pi}{2}}^{\pi}e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$

32. Evaluate the definite integral : $\int_0^{\frac{\pi}{4}}\frac{\sin x+\cos x}{9+16\sin 2x}\,dx$

33. Evaluate the definite integral : $\int_0^1\frac{dx}{\sqrt{1+x}-\sqrt{x}}$

Section C

- **Write the answer of the following questions. [Each carries 4 Marks]** **[56]**

34. Integrate the function : $\dfrac{\sqrt{\tan x}}{\sin x \cdot \cos x}$

35. Find the integral of the function : $\dfrac{1}{\cos(x - a)\cos(x-b)}$

36. Integrate the function : $\dfrac{1}{\sqrt{(x - 1)(x - 2)}}$

37. Integrate the function : $\dfrac{1}{\sqrt{(x - a)(x - b)}}$

38. Integrate the function : $\dfrac{x + 2}{\sqrt{x^2 + 2x + 3}}$

39. Integrate the function : $\dfrac{5x + 3}{\sqrt{x^2 + 4x + 10}}$

40. Integrate the rational function : $\dfrac{2x - 3}{(x^2 - 1)(2x + 3)}$

41. Integrate the rational function : $\dfrac{3x + 5}{x^3 - x^2 - x + 1}$

42. Integrate the rational function : $\dfrac{(x^2 + 1)(x^2 + 2)}{(x^2 + 3)(x^2 + 4)}$

43. Integrate the functions : $\sqrt{1 + 3x - x^2}$

44. Evaluate the integral using substitution : $\displaystyle\int_0^1 \sin^{-1}\left(\frac{2x}{1 + x^2}\right) dx$

45. By using the property of definite integral evaluate the integral in : $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x \; dx}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}$

46. By using the property of definite integral evaluate the integral in : $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$

47. By using the property of definite integral evaluate the integral in : $\displaystyle\int_0^{\pi} \log(1 + \cos x)\, dx$

OPEN STUDENT FOUNDATION
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PRACTICE SHEET DAY 6

Section A

- **Write the answer of the following questions. [Each carries 2 Marks]** **[22]**

1. Find the following integral : $\int x^2\left(1-\frac{1}{x^2}\right)dx$

➥ $I = \int x^2\left(1-\frac{1}{x^2}\right)dx$

$= \int (x^2 - 1)\ dx$

$= \int x^2\ dx - \int x^0\ dx$

$= \frac{x^{2+1}}{2+1} - \frac{x^{0+1}}{0+1} + c$

$= \frac{x^3}{3} - x + c$

2. Find the following integral : $\int \frac{x^3 - x^2 + x - 1}{x - 1}\ dx$

➥ $I = \int \frac{x^3 - x^2 + x - 1}{x - 1}\ dx$

$= \int \frac{x^2(x-1) + 1\ (x-1)}{x-1}\ dx$

$= \int \frac{(x-1)\ (x^2+1)}{(x-1)}\ dx$

$= \int x^2 + 1\ dx$

$= \int x^2\ dx + \int x^0\ dx$

$= \frac{x^{2+1}}{2+1} + \frac{x^{0+1}}{0+1} + c$

$= \frac{x^3}{3} + x + c$

3. Find the following integral : $\int \frac{2 - 3\sin x}{\cos^2 x}\ dx$

➥ $I = \int \frac{2 - 3\sin x}{\cos^2 x}\ dx$

$= \int\left(\frac{2}{\cos^2 x} - \frac{3\sin x}{\cos^2 x}\right) dx$

$= \int (2\sec^2 x - 3\sec x \tan x)\ dx$

$= 2\int (\sec^2 x\ dx - 3\int \sec x\ \tan x\ dx$

$= 2\ \tan x - 3\sec x + c$

4. Integrate the function : $\frac{e^{tan^{-1}x}}{1+x^2}$

➡ $I = \int \frac{e^{\tan^{-1}x}}{1+x^2}\,dx$

Put, $\tan^{-1}x = t \Rightarrow \frac{1}{1+x^2}\,dx = dt$

$I = \int \frac{e^{\tan^{-1}x}}{1+x^2}\,dx$

$= \int e^{t}\,dt$

$= e^t + c$

$= e^{\tan^{-1}x} + c \left(\because t = e^{\tan^{-1}x}\right)$

5. Integrate the function : $\frac{x^3 \sin(\tan^{-1}x^4)}{1+x^8}$

➡ $I = \int \frac{x^3\ \sin(\tan^{-1}x^4)}{1+x^8}\,dx$

Put, $\tan^{-1}x^4 = t \Rightarrow \frac{1}{1+(x^4)^2} \cdot 4x^3\,dx = dt$

$\Rightarrow \frac{x^3}{1+x^8}\,dx = \frac{dt}{4}$

$I = \int \frac{x^3 \sin(\tan^{-1}x^4)}{1+x^8}\,dx$

$= \int \sin t\,\frac{dt}{4}$

$= \frac{-\cos\ t}{4} + c$

$= -\frac{1}{4}\ \cos(\tan^{-1}x^4) + c\ \ (\because\ t = \tan^{-1}\ t^4)$

6. Integrate the function : $\frac{(1+\log x)^2}{x}$

➡ $I = \int \frac{(1+\log x)^2}{x}\,dx$

Put, $1 + \log\ x = t \Rightarrow \frac{1}{x}\,dx = dt$

$= \int t^2\,dt$

$= \frac{t^3}{3} + c$

$= \frac{(1+\log x)^3}{3} + c \qquad (\because\ t = 1 + \log\ x)$

7. Find the integral of the function : $\frac{\sin^3 x + \cos^3 x}{\sin^2 x\ \cos^2 x}$

➥ $I = \int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx$

$= \int \frac{\sin^3 x}{\sin^2 x \cos^2 x} dx + \int \frac{\cos^3 x}{\sin^2 x \cos^2 x} dx$

$= \int \sec x \cdot \tan x \cdot dx + \int \operatorname{cosec} x \cdot \cot x \cdot dx$

$= \sec x - \operatorname{cosec} x + c$

8. Integrate the function : $\frac{4x + 1}{\sqrt{2x^2 + x - 3}}$

➥ $I = \int \frac{4x + 1}{\sqrt{2x^2 + x - 3}} dx$

Let, $2x^2 + x - 3 = t \Rightarrow (4x + 1)\, dx = dt$

$I = \int \frac{dt}{\sqrt{t}}$

$= \int t^{-\frac{1}{2}} dt$

$= \frac{t^{-\frac{1}{2}+1}}{\frac{1}{2}} + c$

$= 2\sqrt{t} + c$

$= 2\sqrt{2x^2 + x - 3} + c \quad (\because\ t = 2x^2 + x - 3)$

9. Integrate the rational function : $\frac{1}{(e^x - 1)}$ (Note : Take $e^x = t$)

➥ $I = \frac{1}{e^x - 1} dx$

Let $e^x = t \Rightarrow e^x dx = dt$

$\Rightarrow dx = \frac{dt}{e^x} = \frac{dt}{t}$

$\therefore I = \int \frac{1}{(t-1)} \cdot \frac{dt}{t}$

$= \int \frac{t-(t-1)}{t(t-1)} dt$

$= \int \left(\frac{1}{t-1} - \frac{1}{t}\right) dt$

$= \int \frac{1}{t-1} dt - \int \frac{1}{t} dt$

$= \log|t-1| - \log|t| + c$

$= \log\left|\frac{t-1}{t}\right| + c$

$= \log\left|\frac{e^x - 1}{e^x}\right| + c \quad (\because\ t = e^x)$

10. Evaluate the definite integral : $\int_0^1 \left(x\, e^x + \sin\frac{\pi x}{4}\right) dx$

➥ $\int_0^1 \left(x\, e^x + \sin\frac{\pi x}{4}\right) dx$

$= \int_0^1 (x\, e^x dx + \int_0^1 \sin\frac{\pi x}{4}\, dx$

$= \left[xe^x - e^x\right]_0^1 + \left[\frac{-\cos\frac{\pi x}{4}}{\frac{\pi}{4}}\right]_0^1$

$\left\{\begin{aligned} &\int xe^x\, dx \\ &= x\int e^x - \int(\frac{d}{dx}(x)\int e^x\, dx)\, dx \\ &= xe^x - \int e^x\, dx \\ &= xe^x - e^x \end{aligned}\right.$

$= (e - e) - (0 - e^0) + \left[-\frac{4}{\pi}\cos\frac{\pi}{4} + \frac{4}{\pi}\cos 0\right]$

$= 1 - \frac{4}{\pi} \times \frac{1}{\sqrt{2}} + \frac{4}{\pi}(1)$

$= 1 + \frac{4}{\pi} \times \frac{2\sqrt{2}}{\pi}$

11. By using the property of definite integral evaluate the integral in : $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^7 x\, dx$

➥ $f(x) = \sin^7 x$

$\therefore\ f(-x) = \sin^7(-x)$

$= -\sin^7 x$

$= -f(x)$

$\therefore\ f(x)$ is an odd function.

$\int_{-a}^{a} f(x) = 0 \quad (\because f(x)$ is an odd function.)

$\therefore\ \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x\, dx = 0$

Section B

- **Write the answer of the following questions. [Each carries 3 Marks]** **[66]**

12. Integrate the function : $\frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}$

➥ $I = \int \frac{2\cos x - 3\sin x}{6\cos x + 4\sin x}\, dx$

$= \int \frac{2\cos x - 3\sin x}{2(3\cos x + 2\sin x)}\, dx$

Put, $3\cos x + 2\sin x = t$

$\therefore\ 2\sin x + 3\cos x = t \Rightarrow (2\cos x - 3\sin x)\, dx = dt\ \ I = \int \frac{2\cos x - 3\sin x}{2(3\cos x + 2\sin x)}\, dx$

$$= \int \frac{dt}{2t}$$

$$= \frac{1}{2}\int \frac{dt}{t}$$

$$= \frac{1}{2} \log |t| + c$$

$$= \frac{1}{2} \log |2 \sin x + 3 \cos x| + c$$

$(\because\ t = 2 \sin x + 3 \cos x)$

13. Integrate the function : $\dfrac{(x+1)(x+\log x)^2}{x}$

➥ $= \displaystyle\int \frac{(x+1)(x+\log x)^2}{x}\, dx$

$= \displaystyle\int \left(\frac{x+1}{x}\right)(x+\log x)^2\, dx$

$= \displaystyle\int \left(1+\frac{1}{x}\right)(x+\log x)^2\, dx$

Put, $x + \log x = t \Rightarrow \left(1+\dfrac{1}{x}\right) dx = dt$

$\text{I} = \displaystyle\int \left(1+\frac{1}{x}\right)(x+\log x)^2\, dx$

$= \int t^2\, dt$

$= \dfrac{t^3}{3} + c$

$= \dfrac{(x+\log x)^3}{3} + c \qquad (\because\ t = x + \log x)$

14. Find the integral of the function : $\dfrac{\cos 2x}{(\cos x + \sin x)^2}$

➥ $\text{I} = \displaystyle\int \frac{\cos 2x}{(\cos x + \sin x)^2}\, dx$

$= \displaystyle\int \frac{\cos^2 x - \sin^2 x}{(\cos x + \sin x)^2}\, dx$

$= \displaystyle\int \frac{(\cos x + \sin x)(\cos x - \sin x)}{(\cos x + \sin x)^2}\, dx$

$= \displaystyle\int \frac{\cos x - \sin x}{\cos x + \sin x}\, dx$

Put, $\cos x + \sin x = t \Rightarrow (-\sin x + \cos x)dx = dt$

$\Rightarrow (\cos x - \sin x)dx = dt$

$\text{I} = \displaystyle\int \frac{1}{t}\, dt$

$= \log |t| + c$

$= \log|\cos x + \sin x| + c \qquad (\because\ t = \cos x + \sin x)$

15. Integrate the function : $\dfrac{1}{\sqrt{(2-x)^2+1}}$

➥ $$I = \int \frac{dx}{\sqrt{(2-x)^2+1}}$$

Put, $2 - x = t \Rightarrow -dx = dt \Rightarrow dx = -dt$

$$I = \int \frac{-dt}{\sqrt{t^2+1}}$$

$$= -\log|t + \sqrt{t^2+1}| + c \qquad \left(\because \int \frac{1}{\sqrt{x^2+a^2}}\,dx = \log\left|x+\sqrt{x^2+a^2}\right| + c\right)$$

$$= -\log\left|(2-x) + \sqrt{(2-x)^2+1}\right| + c$$

$$= \log\left|\frac{1}{(2-x)+\sqrt{(2-x)^2+1}}\right| + c$$

$$= \log\left|\frac{1}{(2-x)+\sqrt{x^2-4x+5}}\right| + c$$

(**NOTE :** It is also solved by taking $2 - x = \tan\theta$ as a substitution.)

16. Integrate the rational function : $\dfrac{1-x^2}{x(1-2x)}$

➥ $$\frac{1-x^2}{x(1-2x)} = \frac{1-x^2}{x-2x^2}$$

Here, the degree of the polynomial in numerator and denominator are equal. So, it is an improper retional functions. we cover it into proper rational functions by dividing process.

$$\therefore \frac{1-x^2}{x-2x^2} = \frac{x^2-1}{2x^2-x}$$

$$= \frac{\frac{1}{2}(2x^2-2)}{2x^2-x}$$

$$= \frac{\frac{1}{2}(2x^2-x+x-2)}{2x^2-x}$$

$$= \frac{1}{2}\left[\frac{2x^2-x}{2x^2-x} + \frac{x-2}{2x^2-x}\right]$$

$$= \frac{1}{2}\left[1 + \frac{x-2}{2x^2-x}\right]$$ which is a proper rational functions.

Now $\dfrac{x-2}{2x^2-x} = \dfrac{x-2}{x(2x-1)} = \dfrac{A}{x} + \dfrac{B}{2x-1}$

$\therefore$ N^r $x - 2 = A(2x-1) + Bx$

$\therefore$ N^r $x - 2 = (2A + B)x - A$

Comparing coefficient of x and constant term on both sides, we get

$\therefore$ $2A + B = 1$ and $-A = -2$

$\therefore$ $A = 2$ and $4 + B = 1 \Rightarrow B = -3$

$$\therefore \frac{x-2}{2x^2 - x} = \frac{2}{x} - \frac{3}{2x-1}$$

$$\therefore \frac{1-x^2}{x(1-2x)} = \frac{1}{2}\left[1 + \frac{2}{x} - \frac{3}{2x-1}\right]$$

$$\therefore \int \frac{1-x^2}{x(1-2x)}\, dx = \int \frac{1}{2}\left[1 + \frac{2}{x} - \frac{3}{2x-1}\right] dx$$

$$= \frac{1}{2}\int dx + \int \frac{1}{x}\, dx - \frac{3}{2}\int \frac{1}{2x-1}\, dx$$

$$= \frac{1}{2}x + \log|x| - \frac{3}{4}\log|2x-1| + c$$

17. Integrate the rational function : $\dfrac{3x-1}{(x-1)(x-2)(x-3)}$

➥ $$\frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

$\therefore$ N^r $3x - 1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-1)(x-2)$

$\therefore$ $3x - 1 = (A + B + C)x^2 - (5A + 4B + 3C)x + 6A + 3B + 2C$

Comparing coefficient of x^2, x and constant term on both sides, we get

$A + B + C = 0$ ....(i)

$-(5A + 4B + 3C) = 3$ ....(ii)

$6A + 3B + 2C = -1$ ....(iii)

(i) $\Rightarrow$ $C = -A - B$

Putting the value of C in (ii) and (iii),

(ii) $\Rightarrow 5A + 4B - 3A - 3B = -3$

$\Rightarrow 2A + B = -3$

(iii) $\Rightarrow 6A + 3B - 2A - 2B = -1$

$\Rightarrow 4A + B = -1$

Now $4A + B = -1$

$2A + B = -3$

$-\quad -\quad +$

Subtract $2A = 2$

$\therefore$ $\boxed{A = 1}$

$4A + B = -1$ and $A = 1$

$\therefore$ $4 + B = -1$

$\therefore$ $\boxed{B = -5}$

$C = -A - B$

$= -1 + 5$

$\therefore$ $\boxed{C = 4}$

$$\therefore \frac{3x-1}{(x-1)(x-2)(x-3)} = \frac{1}{x-1} + \frac{-5}{x-2} + \frac{4}{x-3}$$

$$\therefore \int \frac{3x-1}{(x-1)(x-2)(x-3)}\, dx$$

$$= \int \left(\frac{1}{x-1} - \frac{5}{x-2} + \frac{4}{x-3} \right) dx$$

$$= \int \frac{1}{x-1}\, dx - 5 \int \frac{1}{x-2}\, dx + 4 \int \frac{1}{x-3}\, dx$$

$$= \log |x - 1| - 5 \log |x - 2| + 4\log |x - 3| + c$$

18. Integrate the rational function : $\frac{1}{x(x^4 - 1)}$

➥ $$I = \int \frac{1}{x(x^4 - 1)}\, dx$$

$$= \int \frac{x^3}{x^4(x^4 - 1)}\, dx \quad \text{(Dividing N}^r \text{ and D}^r \text{ by } x^3)$$

Let, $x^4 = t \Rightarrow 4x^3dx = dt$

$$\Rightarrow x^3 dx = \frac{dt}{4}$$

$$\therefore I = \int \frac{dt}{4t(t - 1)}$$

$$= \frac{1}{4} \int \frac{1}{t(t - 1)}\, dt$$

$$= \frac{1}{4} \int \frac{t - (t - 1)}{t(t - 1)}\, dt$$

$$= \frac{1}{4} \int \left(\frac{1}{t - 1} - \frac{1}{t} \right) dt$$

$$= \frac{1}{4} \int \frac{1}{t - 1}\, dt - \frac{1}{4} \int \frac{1}{t}\, dt$$

$$= \frac{1}{4} \log|t - 1| - \frac{1}{4} \log |t| + c$$

$$= \frac{1}{4} \log\left|\frac{t - 1}{t}\right| + c$$

$$= \frac{1}{4} \log\left|\frac{x^4 - 1}{x^4}\right| + c \qquad (\because\ t = x^4)$$

19. Integrate the functions : $\sqrt{1 + \frac{x^2}{9}}$

➥ $$I = \int \sqrt{1 + \frac{x^2}{9}}\, dx$$

$$= \int \frac{1}{3} \sqrt{9 + x^2}\, dx$$

$$= \frac{1}{3} \int \sqrt{(3)^2 + x^2}\, dx$$

Using the formula,

$$\int \sqrt{a^2 + x^2}\, dx = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \log \left| x + \sqrt{a^2 + x^2} \right| + c \text{ we get,}$$

$$I = \frac{1}{3} \int \sqrt{(3)^2 + x^2}\, dx$$

$$= \frac{1}{3}\left[\frac{x}{2}\sqrt{(3)^2 + x^2} + \frac{(3)^2}{2}\log\left|x + \sqrt{(3)^2 + x^2}\right|\right] + c$$

$$= \frac{x}{6}\sqrt{9 + x^2} + \frac{3}{2}\log\left|x + \sqrt{9 + x^2}\right| + c$$

20. Evaluate the integral using substitution : $\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x}\ dx$

➥ Put, $2x = t \Rightarrow 2dx = dt \Rightarrow dx = \frac{dt}{2}$ when $x = 1$ then $t = 2$ and $x = 2$ then $t = 4$

$$\int_1^2 \left(\frac{1}{x} - \frac{1}{2x^2}\right) e^{2x}\ dx = \int_2^4 \left(\frac{2}{t} - \frac{4}{2t^2}\right) e^t\ \frac{dt}{2}$$

$$= \frac{1}{2}\int_2^4 e^t \cdot 2\left(\frac{1}{t} - \frac{1}{t^2}\right) dt$$

$$= \int_2^4 e^t\left(\frac{1}{t} - \frac{1}{t^2}\right) dt$$

$$= \left[e^t \cdot \frac{1}{t}\right]_2^4 \qquad \left(\because \text{Using the formula } \int e^x\ (f(x) + f(x))\ dx = f(x)e^x + c\right)$$

$$= \frac{e^4}{4} - \frac{e^2}{2} = \frac{e^2}{2}\left(\frac{e^2}{2} - 1\right)$$

$$= e^2\ \frac{(e^2 - 2)}{4}$$

21. By using the property of definite integral evaluate the integral in : $\int_0^{\pi} \frac{x\ dx}{1 + \sin x}$

➥ $$\mathrm{I} = \int_0^{\pi} \frac{x\ dx}{1 + \sin x} \qquad \text{....(i)}$$

$$\mathrm{I} = \int_0^{\pi} \frac{(\pi - x)\ dx}{1 + \sin\ (\pi - x)} \qquad \left\{\because \int_0^a f(x)dx = \int_0^a f(a - x)dx\right.$$

$$\mathrm{I} = \int_0^{\pi} \frac{(\pi - x)dx}{1 + \sin x} \qquad \text{....(ii)}$$

Adding equation (i) and (ii), we get,

$$2\mathrm{I} = \int_0^{\pi} \frac{xdx}{1 + \sin x} + \int_0^{\pi} \frac{(\pi - x)dx}{1 + \sin x}$$

$$= \int_0^{\pi} \frac{(x + \pi - x)dx}{1 + \sin x}$$

$$= \int_0^{\pi} \frac{\pi}{1 + \sin x}\ dx$$

$$= \pi \int_0^{\pi} \frac{1}{1 + \sin x} \times \frac{1 - \sin x}{1 - \sin x}\ dx$$

$$= \pi \int_0^{\pi} \frac{1 - \sin x}{1 - \sin^2 x}\, dx$$

$$= \pi \int_0^{\pi} \frac{1 - \sin x}{\cos^2 x}\, dx$$

$$= \pi \int_0^{\pi} (\sec^2 x - \sec x \tan x)\, dx$$

$$= \pi \left[\tan x - \sec x\right]_0^{\pi}$$

$$2I = \pi\ [(\tan \pi - \sec \pi) - (\tan 0 - \sec 0)] = [(0 - (-1) - 0 + 1]$$

$$= 2\pi$$

$$\therefore\ I = \pi$$

22. By using the property of definite integral evaluate the integral in : $\int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a - x}}\, dx$

➥ $I = \int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a - x}}\, dx$ ....(i)

$$I = \int_0^a \frac{\sqrt{a - x}}{\sqrt{a - x} + \sqrt{a - a + x}}\, dx$$

$I = \int_0^a \frac{\sqrt{a - x}}{\sqrt{a - x} + \sqrt{x}}\, dx$ ....(ii)

Adding results (i) and (ii), we get,

$$2I = \int_0^a \frac{\sqrt{x}}{\sqrt{x} + \sqrt{a - x}}\, dx + \int_0^a \frac{\sqrt{a - x}}{\sqrt{a - x} + \sqrt{x}}\, dx$$

$$= \int_0^a \frac{\sqrt{x} + \sqrt{a - x}}{\sqrt{x} + \sqrt{a - x}}\, dx$$

$$= \int_0^a dx$$

$$= [x]_0^a$$

$$= a$$

$$\therefore\ I = \frac{a}{2}$$

23. Integrate the function : $f'(ax + b)\ [f(ax + b)]^n$

➥ $I = \int f'(ax + b)\ [f(ax + b)]^n\, dx$

Put, $f(ax + b) = t$

$$\therefore\ f'(ax + b) \cdot a\, dx = dt$$

$$\therefore\ f'(ax + b) \cdot dx = \frac{dt}{a}$$

$$I = \int t^n \cdot \frac{dt}{a}$$

$$= \frac{1}{a} \cdot \frac{t^{n+1}}{n+1} + c$$

$$= \frac{[f(ax+b)]^{n+1}}{a(n+1)} + c$$

24. Integrate the function : $\frac{1}{(x^2+1)(x^2+4)}$

➥ $$\int \frac{1}{(x^2+1)(x^2+4)} dx = \frac{1}{3} \int \frac{(x^2+4)-(x^2+1)}{(x^2+4)(x^2+1)} dx$$

$$= \frac{1}{3} \int \left(\int \frac{1}{x^2+1} - \frac{1}{x^2+4} \right) dx$$

$$= \frac{1}{3} \left[\tan^{-1} x - \frac{1}{2} \tan^{-1} \frac{x}{2} \right] + c$$

$$= \frac{1}{3} \tan^{-1} x - \frac{1}{6} \tan^{-1} \frac{x}{2} + c$$

25. Integrate the function : $\frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x}$

➥ $$\sin^8 x - \cos^8 x = (\sin^4 x - \cos^4 x)(\sin^4 x + \cos^4 x)$$

$$= (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)[(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x]$$

$$= (1)(-\cos 2x)(1 - 2\sin^2 x \cos^2 x)$$

$$\therefore \int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} = -\cos 2x$$

$$\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} dx = -\int \cos 2x \, dx$$

$$= -\frac{\sin 2x}{2} + c$$

26. Integrate the function : $\frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}}$

➥ $$I = \int \frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}} dx$$

$$= \int \frac{x^5 - x^4}{x^3 - x^2} dx$$

$$= \int \frac{x^4 (x-1)}{x^2 (x-1)} dx$$

$$= \int x^2 \, dx$$

$$= \frac{x^3}{3} + c$$

27. Integrate the function : $\frac{5x}{(x+1)(x^2+9)}$

➥ $$\frac{5x}{(x+1)(x^2+9)} = \frac{A}{x+1} + \frac{Bx + C}{x^2+9} \qquad .....(i)$$

N^r $5x = A(x^2 + 9) + (Bx + C)(x + 1)$

$5x = (A + B)x^2 + (B + C)x + 9A + C$

Comparing the coefficient of x^2, x and constant terms on both sides, we get

$A + B = 0 \Rightarrow B = -A$

$B + C = 5 \Rightarrow -A + C = 5$

$9A + C = 0 \Rightarrow 9A + C = 0$

$\quad - \quad -$

Subtract $-10A = 5 \Rightarrow A = -\frac{1}{2}$

$B = -A \Rightarrow B = \frac{1}{2}$

$C = 5 - B = 5 - \frac{1}{2} = \frac{9}{2}$

From (i) $\Rightarrow$

$$\therefore \int \frac{5x}{(x+1)(x^2+9)}\,dx = \int\left(\frac{-\frac{1}{2}}{x+1} + \frac{\frac{1}{2}x + \frac{9}{2}}{x^2+9}\right)dx$$

$$= -\frac{1}{2}\int\frac{1}{x+1}\,dx + \frac{1}{4}\int\frac{2x}{x^2+9}\,dx + \frac{9}{2}\int\frac{1}{x^2+(3)^2}\,dx$$

$$= -\frac{1}{2}\log|x+1| + \frac{1}{4}\log|x^2+9| + \frac{9}{2}\times\frac{1}{3}\tan^{-1}\frac{x}{3} + c$$

$$= -\frac{1}{2}\log|x+1| + \frac{1}{4}\log|x^2+9| + \frac{3}{2}\tan^{-1}\left(\frac{x}{3}\right) + c$$

28. Integrate the function : $\frac{1}{x^{\frac{1}{2}} + x^{\frac{1}{3}}}$ $\left[\text{Hint : } \frac{1}{x^{\frac{1}{2}} + x^{\frac{1}{3}}} = \frac{1}{x^{\frac{1}{3}}\left(1 + x^{\frac{1}{6}}\right)}, \text{ Put } x = t^6\right]$

➥ $I = \int \frac{1}{x^{\frac{1}{2}} + x^{\frac{1}{3}}}\,dx$

L.C.M. of 2 and 3 is 6. $x = t^6 \Rightarrow dx = 6t^5\,dt$

$$I = \int \frac{1}{(t^6)^{\frac{1}{2}} + (t^6)^{\frac{1}{3}}}\cdot 6t^5\,dt$$

$$= \int\frac{6t^5}{t^3+t^2}\,dt$$

$$= \int\frac{6t^5}{t^2(t+1)}\,dt$$

$$= 6\int\frac{t^3+1-1}{t+1}\,dt$$

$$= 6\int\left(\frac{t^3+1}{t+1} - \frac{1}{t+1}\right)dt$$

$$= 6\int\left(\frac{(t+1)(t^2-t+1)}{t+1} - \frac{1}{t+1}\right)dt$$

$$= 6 \int \left(t^2 - t + 1 - \frac{1}{t+1} \right) dt$$

$$I = 6 \left[\frac{t^3}{3} - \frac{t^2}{2} + t - \log |t+1| \right] + c$$

$$= 2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6 \log \left| x^{\frac{1}{6}} + 1 \right| + c \qquad \left(\because t = x^{\frac{1}{6}} \right)$$

29. Evaluate the definite integral : $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x \, dx}{\cos^2 x + 4\sin^2 x}$

➥ $I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x \, dx}{\cos^2 x + 4\sin^2 x}$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos^2 x \, dx}{\cos^2 x + 4(1 - \cos^2 x)}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos^2 x \, dx}{4 - 3\cos^2 x}$$

$$= -\frac{1}{3} \int_0^{\frac{\pi}{2}} \frac{4 - 3\cos^2 x - 4}{4 - 3\cos^2 x} \, dx$$

$$= -\frac{1}{3} \int_0^{\frac{\pi}{2}} \left(1 - \frac{4}{4 - 3\cos^2 x} \right) dx$$

$$= -\frac{1}{3} \int_0^{\frac{\pi}{2}} dx + \frac{4}{3} \int_0^{\frac{\pi}{2}} \frac{1}{4 - 3\cos^2 x} \, dx$$

$$= -\frac{1}{3} [x]_0^{\frac{\pi}{2}} + \frac{4}{3} I_1$$

$$= -\frac{\pi}{6} + \frac{4}{3} I_1 \qquad \text{....(i)}$$

where $I_1 = \int_0^{\frac{\pi}{2}} \frac{1}{4 - 3\cos^2 x} \, dx$

Divide numerator and denominator by $\cos^2 x$ we get,

$$I_1 = \int_0^{\frac{\pi}{2}} \frac{\sec^2 x \, dx}{4\sec^2 x - 3}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sec^2 x \, dx}{4(1 + \tan^2 x) - 3}$$

π

$$= \int_0^2 \frac{\sec^2 x \, dx}{4 \tan^2 x + 1}$$

Let $\tan x = t \Rightarrow \sec^2 x \, dx = dt$

$x = 0 \Rightarrow t = 0$ and $x = \frac{\pi}{2} \Rightarrow t = \infty$

$$I_1 = \int_0^\infty \frac{dt}{4t^2 + 1}$$

$$= \frac{1}{4} \int_0^\infty \frac{dt}{t^2 + \left(\frac{1}{2}\right)^2}$$

$$= \frac{1}{4} \cdot \frac{1}{\frac{1}{2}} \left[\tan^{-1} 2t\right]_0^\infty$$

$$= \frac{1}{2} \left[\tan^{-1} \infty - \tan^{-1} 0\right]$$

$$= \frac{1}{2} \left[\frac{\pi}{2}\right] = \frac{\pi}{4}$$

Substitute the value of I_1 in result (i), we have

$$I = -\frac{\pi}{6} + \frac{4}{3} \cdot \frac{\pi}{4} = -\frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{6}$$

30. Integrate the function : $\dfrac{\sqrt{x^2 + 1}\left[\log(x^2 + 1) - 2\log x\right]}{x^4}$

➥ $$I = \int \frac{\sqrt{x^2 + 1}\left[\log(x^2 + 1) - 2\log x\right]}{x^4}$$

$$= \int \frac{x\sqrt{1 + \frac{1}{x^2}}\left[\log\left(\frac{x^2 + 1}{x^2}\right)\right]}{x^4} dx$$

$$= \int \frac{\sqrt{1 + \frac{1}{x^2}}\left[\log\left(1 + \frac{1}{x^2}\right)\right]}{x^3} dx$$

Put, $1 + \frac{1}{x^2} = t \Rightarrow \frac{-2}{x^3} dx = dt$

$$\Rightarrow \frac{1}{x^3} dx = -\frac{dt}{2}$$

$$I = -\frac{1}{2} \int \sqrt{t} \cdot \log t \, dt$$

Now using the rule of integration by parts we get,

$$I = -\frac{1}{2}\left[\log t \int \sqrt{t} \, dt - \int \left(\frac{d}{dt} (\log t) \int \sqrt{t} \, dt\right) dt\right]$$

$$I = -\frac{1}{2}\left[\log t \, \frac{2}{3} t^{\frac{3}{2}} - \int \frac{1}{t} \cdot \frac{2}{3} t^{\frac{3}{2}} \, dt\right]$$

$$= -\frac{1}{2}\left[\log t \cdot \frac{2}{3} \cdot t^{\frac{3}{2}} - \frac{2}{3}\int t^{\frac{1}{2}}\, dt\right]$$

$$= -\frac{1}{2}\left[\log t\ \frac{2}{3}\, t^{\frac{3}{2}} - \frac{2}{3} \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right] + c$$

$$= -\frac{1}{3}\log t \cdot t^{\frac{3}{2}} + \frac{2}{9}\, t^{\frac{3}{2}} + c$$

$$= -\frac{1}{3}\, t^{\frac{3}{2}}\left[\log t - \frac{2}{3}\right] + c$$

$$= -\frac{1}{3}\left[1 + \frac{1}{x^2}\right]^{\frac{3}{2}}\left[\log\left(1 + \frac{1}{x^2}\right) - \frac{2}{3}\right] + c$$

31. Evaluate the definite integral : $\int_{\frac{\pi}{2}}^{\pi} e^x\left(\frac{1 - \sin x}{1 - \cos x}\right) dx$

➥ $I = \int_{\frac{\pi}{2}}^{\pi} e^x\left(\frac{1 - \sin x}{1 - \cos x}\right) dx$

$$= \int_{\frac{\pi}{2}}^{\pi} e^x\left(\frac{1 - 2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}\right) dx$$

$$I = \int_{\frac{\pi}{2}}^{\pi} e^x\left(\frac{1}{2}\text{cosec}^2\frac{x}{2} - \cot\frac{x}{2}\right) dx$$

$$= -\int_{\frac{\pi}{2}}^{\pi} e^x\left(\cot\frac{x}{2} - \frac{1}{2}\text{cosec}^2\frac{x}{2}\right) dx$$

$$= \left[-e^x \cot\frac{x}{2}\right]_{\frac{\pi}{2}}^{\pi} \quad \left\{\begin{array}{l} \because f(x) = \cot\frac{x}{2} \\ f'(x) = -\frac{1}{2}\text{cosec}^2 x \\ \int e^x(f(x) + f'(x)\, dx = e^x\ f(x) + c \end{array}\right.$$

$$= -\left[e^{\pi}\cot\frac{\pi}{2} - e^{\frac{\pi}{2}}\cot\frac{\pi}{4}\right]$$

$$= -\left[0 - e^{\frac{\pi}{2}}\right]$$

$$= e^{\frac{\pi}{2}}$$

32. Evaluate the definite integral : $\int^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16\sin 2x}\, dx$

0

➥ $I = \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x}\, dx$

Let $\sin x - \cos x = t \Rightarrow (\cos x + \sin x)\ dx = dt$ $x = 0$ then $t = \sin 0 - \cos 0 = -1$

$x = \frac{\pi}{4}$ then $t = \sin \frac{\pi}{4} - \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0$

Also $\sin x - \cos x = t$

$\therefore\ (\sin x - \cos x)^2 = t^2$

$\therefore\ \sin^2 x + \cos^2 x - 2 \sin\ \cos x = t^2$

$\therefore\ 1 - \sin 2x = t^2$

$\therefore\ \sin 2x = 1 - t^2$

$I = \int_{-1}^{0} \frac{dt}{9 + 16\,(1 - t^2)}$

$= \int_{-1}^{0} \frac{dt}{25 - 16t^2}$

$= \frac{1}{16} \int_{-1}^{0} \frac{dt}{\left(\frac{5}{4}\right)^2 - t^2}$

$= \frac{1}{16} \times \frac{1}{2\left(\frac{5}{4}\right)} \left[\log \left| \frac{\frac{5}{4} + t}{\frac{5}{4} - t} \right| \right]_{-1}^{0}$

$= \frac{1}{40} \left[\log \left| \frac{5 + 4t}{5 - 4t} \right| \right]_{-1}^{0}$

$= \frac{1}{40} \left[\log 1 - \log \frac{1}{9} \right]$

$= \frac{1}{40} [0 + \log 9] = \frac{1}{40} \log 9$

33. Evaluate the definite integral : $\int_0^1 \frac{dx}{\sqrt{1 + x} - \sqrt{x}}$

➥ $I = \int_0^1 \frac{dx}{\sqrt{1 + x} - \sqrt{x}}$

$= \int_0^1 \frac{\sqrt{1 + x} + \sqrt{x}}{\left(\sqrt{1 + x} - \sqrt{x}\right)\left(\sqrt{1 + x} + \sqrt{x}\right)}\, dx$

$I = \int_0^1 \frac{\sqrt{1+x} + \sqrt{x}}{1 + x - x}\, dx$

$= \int_0^1 \left((1 + x)^{\frac{1}{2}} - x^{\frac{1}{2}} \right) dx$

$$= \int_0^1 (1+x)^{\frac{1}{2}}\, dx - \int_0^1 x^{\frac{1}{2}}\, dx$$

$$= \frac{2}{3}\left[(1+x)^{\frac{3}{2}}\right]_0^1 - \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^1$$

$$= \frac{2}{3}\left[2^{\frac{3}{2}} - 1\right] - \frac{2}{3}[1]$$

$$= \frac{2}{3}\left(2\sqrt{2} - 1 + 1\right) = \frac{4\sqrt{2}}{3}$$

Section C

- **Write the answer of the following questions. [Each carries 4 Marks]** **[56]**

34. Integrate the function : $\dfrac{\sqrt{\tan x}}{\sin x \cdot \cos x}$

➥ $I = \int \dfrac{\sqrt{\tan x}}{\sin x \cdot \cos x}\, dx$

$$= \int \frac{\sqrt{\tan x}}{\frac{\sin x}{\cos x} \cdot \cos^2 x}\, dx$$

$$= \int \frac{\sqrt{\tan x}}{\tan x} \cdot \sec^2 x\, dx$$

$$= \int \frac{1}{\sqrt{\tan x}} \cdot \sec^2 x\, dx$$

Put, $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$$I = \int \frac{1}{\sqrt{t}} \cdot dt$$

$$= \int \frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$$

$$= 2\sqrt{t} + c$$

$$= 2\sqrt{\tan x} + c \qquad (\because\ t = \tan x)$$

35. Find the integral of the function : $\dfrac{1}{\cos(x-a)\cos(x-b)}$

➥ $I = \int \dfrac{1}{\cos(x-a)\cos(x-b)}\, dx$

$$= \frac{1}{\sin(a-b)} \int \frac{\sin[(x-b)-(x-a)]}{\cos(x-a)\cos(x-b)}\, dx$$

$$= \frac{1}{\sin(a-b)} \int \frac{\sin(x-b)\cos(x-a) - \cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)}\, dx$$

$$= \frac{1}{\sin(a-b)} \int [\tan(x-b) - \tan(x-a)]\, dx$$

$$= \frac{1}{\sin(a-b)} [\int \tan(x-b)\, dx - \int \tan(x-a)\, dx]$$

$$= \frac{1}{\sin(a-b)} [\log|\sec(x-b)| - \log|\sec(x-a)|] + c$$

$$= \frac{1}{\sin(a-b)} \log\left|\frac{\sec(x-b)}{\sec(x-a)}\right| + c$$

$$= \frac{1}{\sin(a-b)} \log\left|\frac{\cos(x-a)}{\cos(x-b)}\right| + c$$

36. Integrate the function : $\frac{1}{\sqrt{(x-1)(x-2)}}$

➥ $$I = \int \frac{1}{\sqrt{(x-1)(x-2)}}\, dx$$

$$= \int \frac{1}{\sqrt{x^2 - 3x + 2}}\, dx$$

$$= \int \frac{1}{\sqrt{x^2 - 3x + \frac{9}{4} - \frac{9}{4} + 2}}\, dx \quad \left(\because \text{L.T.} = \frac{(\text{M.T.})^2}{4 \times \text{F.T.}} \text{ to make it perfect square}\right)$$

$$= \int \frac{1}{\sqrt{\left(x - \frac{3}{2}\right)^2 - \frac{1}{4}}}\, dx$$

$$= \int \frac{1}{\sqrt{\left(x - \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}\, dx$$

$$I = \log\left|x - \frac{3}{2} + \sqrt{\left(x - \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2}\right| + c \quad \left(\because \int \frac{1}{x^2 - a^2}\, dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c\right)$$

$$I = \log\left|x - \frac{3}{2} + \sqrt{x^2 - 3x + \frac{9}{4} - \frac{1}{4}}\right| + c$$

$$= \log\left|x - \frac{3}{2} + \sqrt{x^2 - 3x + 2}\right| + c$$

37. Integrate the function : $\frac{1}{\sqrt{(x-a)(x-b)}}$

➥ $$I = \int \frac{1}{\sqrt{(x-a)(x-b)}}\, dx$$

$$= \int \frac{1}{\sqrt{x^2 - (a+b)x + ab}}\, dx$$

$$= \int \frac{dx}{\sqrt{x^2 - (a+b)x + \left(\frac{a+b}{2}\right)^2 - \left(\frac{a+b}{2}\right)^2 + ab}}$$ ($\because$ To make perfect square, add and subtract Last term)

$$= \int \frac{dx}{\sqrt{\left(x - \frac{a+b}{2}\right)^2 - \left(\frac{(a+b)^2}{4} - ab\right)}}$$

$$= \int \frac{dx}{\sqrt{\left(x - \frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2}}$$

$$= \log \left| \left(x - \frac{a+b}{2}\right) + \sqrt{\left(x - \frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2} \right| + c \quad \left(\because \int \frac{dx}{\sqrt{x^2 - a^2}} \log \left| x + \sqrt{x^2 - a^2} \right| + c\right)$$

$$= \log \left| \left(x - \frac{a+b}{2}\right) + \sqrt{(x-a)(x-b)} \right| + c$$

38. Integrate the function : $\frac{x+2}{\sqrt{x^2 + 2x + 3}}$

➥ $I = \int \frac{x+2}{\sqrt{x^2 + 2x + 3}} dx$

$$I = \int \frac{\frac{1}{2}(2x+4)}{\sqrt{x^2 + 2x + 3}} dx$$

$$= \int \frac{\frac{1}{2}(2x+2+2)}{\sqrt{x^2 + 2x + 3}} dx$$

$$= \int \frac{\frac{1}{2}(2x+2) + \frac{1}{2}(2)}{\sqrt{x^2 + 2x + 3}} dx$$

$$= \frac{1}{2} \int \frac{2x+2}{\sqrt{x^2 + 2x + 3}} dx + \int \frac{1}{\sqrt{x^2 + 2x + 3}} dx$$

$$= \frac{1}{2} \int \frac{\frac{d}{dx}(x^2 + 2x + 3)}{\sqrt{x^2 + 2x + 3}} dx + \int \frac{1}{\sqrt{x^2 + 2x + 1 + 2}} dx$$ ($\because$ To make it perfect square)

$$= \frac{1}{2} \int \frac{\frac{d}{dx}(x^2 + 2x + 3)}{\sqrt{x^2 + 2x + 3}} dx + \int \frac{1}{\sqrt{(x+1)^2 + (\sqrt{2})2}} dx$$

Now $\int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + c$ and $\int \frac{1}{x^2 + a^2} dx = \log \left| x + \sqrt{x^2 + a^2} \right| + c$

Using these both formulae, we get

$$I = \frac{1}{2} \times 2\sqrt{x^2 + 2x + 3} + \log \left| (x+1) + \sqrt{(x+1)^2 + (\sqrt{2})^2} \right| + c$$

$$= \sqrt{x^2+2x+3} + \log\left|(x+1)+\sqrt{x^2+2x+3}\right| + c$$

39. Integrate the function : $\dfrac{5x+3}{\sqrt{x^2+4x+10}}$

➥ $I = \int \dfrac{5x+3}{\sqrt{x^2+4x+10}}\, dx$

$5x + 3 = A\dfrac{d}{dx}(x^2+4x+10) + B$

$\therefore\ 5x+3 = A(2x+4) + B$ ....(i)

$\therefore\ 5x+3 = 2A \cdot x + 4A + B$

Equating the coefficient of x from both sides,

we get $2A = 5 \Rightarrow A = \dfrac{5}{2}$

Equating the constant terms from both sides,

we get $4A + B = 3$ but $A = \dfrac{5}{2}$ then $4\left(\dfrac{5}{2}\right) + B = 3$

$\therefore\ B = 3 - 10 = -7$

Substituting the value of A and B in (i), we have

$5x + 3 = \dfrac{5}{2}(2x+4) - 7$

$I = \int \dfrac{5x+3}{\sqrt{x^2+4x+10}}\, dx$

$= \int \dfrac{\frac{5}{2}(2x+4) - 7}{\sqrt{x^2+4x+10}}\, dx$

$= \dfrac{5}{2}\int \dfrac{2x+4}{\sqrt{x^2+4x+10}}\, dx - 7\int \dfrac{1}{\sqrt{x^2+4x+10}}\, dx$

$= \dfrac{5}{2} I_1 - 7I_2$ ....(ii)

$I_1 = \int \dfrac{2x+4}{\sqrt{x^2+4x+10}}\, dx$

Let, $x^2 + 4x + 10 = t \Rightarrow (2x+4)\, dx = dt$

$\therefore\ I_1 = \int \dfrac{dt}{\sqrt{t}} = \int t^{-\frac{1}{2}}\, dt$

$\therefore\ I_1 = 2\sqrt{t} + c_1$

$= 2\sqrt{x^2+4x+10} + c_1$ ($\because$ Put, the value of t)

$I_2 = \int \dfrac{1}{\sqrt{x^2+4x+10}}\, dx$

$= \int \dfrac{1}{\sqrt{x^2+4x+4+6}}\, dx$ ($\because$ To make perfect square)

$$= \int \frac{1}{\sqrt{(x+2)^2 + (\sqrt{6})^2}}\, dx$$

$$= \log\left|(x+2) + \sqrt{(x+2)^2 (\sqrt{6})^2}\right| + c_2 \qquad \left(\because \int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x + \sqrt{x^2+a^2}\right| + c\right)$$

$$= \log\left|(x+2) + \sqrt{x^2 + 4x\ 10}\right| + c_2$$

Putting the values of I_1 and I_2 in (ii), we get

$$I = \frac{5}{2} \times 2\sqrt{x^2+4x+10} + c_1 - 7\log\left|(x+2) + \sqrt{x^2+4x+10}\right| + c_2$$

$$= 5\sqrt{x^2+4x+10} - 7\log\left|(x+2) + \sqrt{x^2+4x+10}\right| + c$$

where $c_1 + c_2 = c$ = constant

40. Integrate the rational function : $\dfrac{2x-3}{(x^2-1)(2x+3)}$

➡ $\dfrac{2x-3}{(x^2-1)(2x+3)} = \dfrac{2x-3}{(x-1)(x+1)(2x+3)}$

$\therefore \dfrac{2x-3}{(x^2-1)(2x+3)} = \dfrac{A}{(x-1)} + \dfrac{B}{(x+1)} + \dfrac{C}{2x+3}$ ...(1)

$N^r\ 2x - 3 = A(x+1)(2x+3) + B(x-1)(2x+3) + C(x-1)(x+1)$

$\therefore 2x - 3 = (2A + 2B + C)x^2 + (5A + B)x + (3A - 3B - C)$

Comparing coefficient of x^2, x and constant term on both sides, we get

$2A + 2B + C = 0$ ....(i)

$5A + B = 2$ ....(ii)

$3A - 3B - C = -3$ ....(iii)

(ii) $\Rightarrow B = 2 - 5A$

Put, the value of B in equation (i) and (ii),

(i) $\Rightarrow 2A + 4 - 10A + C = 0 \Rightarrow -8A + C = -4$

(iii)$\Rightarrow 3A - 6 + 15A - C = -3 \Rightarrow \underline{18A - C = 3}$

Add : $10A = -1$

$$\therefore \boxed{A = -\frac{1}{10}}$$

$B = 2 - 5A$ and $A = -\dfrac{1}{10} \Rightarrow B = 2 + \dfrac{1}{2} = \dfrac{5}{2}$

$$\therefore \boxed{B = \frac{5}{2}}$$

(i) $\Rightarrow 2A + 2B + C = 0$

$\therefore C = -2A - 2B$ and $A = -\dfrac{1}{10}$ and $B = \dfrac{5}{2}$

$\therefore C = \dfrac{1}{5} - 5 = -\dfrac{24}{5}$ $\qquad \therefore \boxed{C = -\frac{24}{5}}$

Substitute the values of A, B and C in (1), we get,

$$\frac{2x-3}{(x^2-1)(2x+3)} = \frac{-\frac{1}{10}}{x-1} + \frac{\frac{5}{2}}{x+1} + \frac{-\frac{24}{5}}{2x+3}$$

$$\therefore \int \frac{2x-3}{(x^2-1)(2x+3)}\,dx$$

$$= \int\left(\int \frac{-1}{10(x-1)} + \frac{5}{2(x+1)} - \frac{24}{5(2x+3)}\right)dx$$

$$= -\frac{1}{10}\int\frac{dx}{x-1} + \frac{5}{2}\int\frac{dx}{x+1} - \frac{24}{5}\int\frac{dx}{2x+3}$$

$$= -\frac{1}{10}\log|x-1| + \frac{5}{2}\log|x+1| - \frac{24}{5}\cdot\frac{1}{2}\log|2x+3| + c$$

$$= \frac{5}{2}\log|x+1| - \frac{1}{10}\log|x-1| - \frac{12}{5}\cdot\log|2x+3| + c$$

41. Integrate the rational function : $\frac{3x+5}{x^3-x^2-x+1}$

➥ $$\frac{3x+5}{x^3-x^2-x+1} = \frac{3x+5}{x^2(x-1)-1(x-1)}$$

$$= \frac{3x+5}{(x-1)(x^2-1)}$$

$$= \frac{3x+5}{(x-1)(x-1)(x+1)}$$

$$= \frac{3x+5}{(x-1)^2(x+1)}$$

$$\therefore \frac{3x+5}{x^3-x^2-x+1} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1} \quad ...(1)$$

$\therefore$ N^r $3x+5 = A(x-1)(x+1) + B(x+1) + C(x-1)^2$

$\therefore$ $3x+5 = (A+C)x^2 + (B-2C)x + (-A+B+C)$

Comparing coefficient of x^2, x and constant term on both sides, we get

$A + C = 0$ ....(i)

$B - 2C = 3$ ....(ii)

$-A + B + C = 5$ ....(iii)

(i) $\Rightarrow A + C = 0 \Rightarrow A = -C$

(iii) $\Rightarrow C + B + C = 5$

$\therefore B + 2C = 5$	(ii) $B - 2C = 3$	$A = -C$ and
(ii) $\Rightarrow \underline{B - 2C = 3}$	and $B = 4$	$C = \frac{1}{2}$
Add $2B = 8$	$4 - 2C = 3$	
$\therefore \boxed{B = 4}$	$\therefore \boxed{C = \frac{1}{2}}$	$\therefore \boxed{A = -\frac{1}{2}}$

Substitute the values of A, B and C in (1), we get

$$\frac{3x+5}{x^3-x^2-x+1} = \frac{-1}{2(x-1)} + \frac{4}{(x-1)^2} + \frac{1}{2(x+1)}$$

$$\therefore \int \frac{3x+5}{x^3-x^2-x+1}\,dx$$

$$= \int\left(\frac{-1}{2(x-1)} + \frac{4}{(x-1)^2} + \frac{1}{2(x+1)}\right)dx$$

$$= -\frac{1}{2}\int\frac{dx}{(x-1)} + 4\int\frac{dx}{(x-1)^2} + \frac{1}{2}\int\frac{dx}{x+1}$$

$$= -\frac{1}{2}\log|x-1| + 4\frac{(x-1)^{-2+1}}{-1} + \frac{1}{2}\log + |x+1| + C$$

$$= \frac{1}{2}\log\left|\frac{x+1}{x-1}\right| \frac{-4}{(x-1)} + c$$

42. Integrate the rational function : $\dfrac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}$

➥ $\dfrac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}$ Taking $x^2 = t$ (Not substitution)

$$\frac{(t+1)(t+2)}{(t+3)(t+4)} = \frac{t^2+3t+2}{t^2+7t+12}$$

Here, the degree of polynomial in numerator and denominator is equal. so, we first do the division process.

$$\therefore \frac{(t+1)(t+2)}{(t+3)(t+4)} = \frac{(t^2+7t+12)-4t-10}{t^2+7t+12}$$

$$= 1 - \frac{(4t+10)}{t^2+7t+12}$$

$$= 1 - \frac{4t+10}{(t+3)(t+4)}$$

Now use $\dfrac{4t+10}{(t+3)(t+4)}$

$$\frac{4t+10}{(t+3)(t+4)} = \frac{A}{t+3} + \frac{B}{t+4}$$

N^r $4t + 10 = A(t+4) + B(t+3)$

$\therefore$ $4t + 10 = (A + B)\,t + 4A + 3B$

Comparing coefficient of t and constant term on both sides, we get

$A + B = 4$ ....(i)

$4A + 3B = 10$ ....(ii)

(i) $\times 4 \Rightarrow 4A + 4B = 16$ | $A + B = 4$ and $B = 6$

(i) $\Rightarrow$ $4A + 3B = 10$ | $\therefore A = -2$

$\quad - \quad - \quad\quad -$

Subtract B = 6

$$\therefore \frac{4t + 10}{(t+3)(t+4)} = \frac{-2}{t+3} + \frac{6}{t+4}$$

$$\therefore \frac{(t+1)(t+2)}{(t+3)(t+4)} = 1 - \left[\frac{-2}{t+3} + \frac{6}{t+4}\right]$$

$$= 1 + \frac{2}{t+3} - \frac{6}{t+4}$$

Now Put, $t = x^2$

$$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = 1 + \frac{2}{x^2+3} - \frac{6}{x^2+4}$$

$$\therefore \int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} dx = \int \left[1 + \frac{2}{x^2+3} - \frac{6}{x^2+4}\right] dx$$

$$= \int dx + 2\int \frac{dx}{x^2 + (\sqrt{3})^2} - 6\int \frac{dx}{x^2 + (2)^2}$$

$$= x + \frac{2}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} - \frac{6}{2} \tan^{-1} \frac{x}{2} + c$$

$$= x + \frac{2}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} - 3\tan^{-1} \frac{x}{2} + c$$

43. Integrate the functions : $\sqrt{1 + 3x - x^2}$

➥ $I = \int \sqrt{1 + 3x - x^2}\, dx$

$$= \int \sqrt{1 - (x^2 - 3x)}\, dx$$

$$= \int \sqrt{1 + \frac{9}{4} - \left(x^2 - 3x + \frac{9}{4}\right)}\, dx \quad (\because \text{ Making perfect square})$$

$$= \int \sqrt{\frac{13}{4} - \left(x - \frac{3}{2}\right)^2}\, dx$$

$$= \int \sqrt{\left(\frac{\sqrt{13}}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2}\, dx$$

Using the formula,

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + c$$

we get,

$$I = \int \sqrt{\left(\frac{\sqrt{13}}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2}\, dx$$

$$= \frac{x - \frac{3}{2}}{2} \sqrt{\left(\frac{\sqrt{13}}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2} + \frac{\left(\frac{\sqrt{13}}{2}\right)^2}{2} \sin^{-1} \frac{\left(x - \frac{3}{2}\right)}{\frac{\sqrt{13}}{2}} + c$$

$$= \frac{2x - 3}{2}\sqrt{1 + 3x - x^2} + \frac{13}{8} \sin^{-1}\left(\frac{2x - 3}{\sqrt{13}}\right) + c$$

44. Evaluate the integral using substitution : $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$

➥ Put $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

when $x = 0$ then $\theta = 0$ and $x = 1$ then $\theta = \frac{\pi}{4}$

$$= \int_0^{\frac{\pi}{4}} \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) \cdot \sec^2\theta . d\theta$$

$$= \int_0^{\frac{\pi}{4}} \sin^{-1}(\sin 2\theta) \cdot \sec^2\theta . d\theta$$

$$= \int_0^{\frac{\pi}{4}} 2\theta \cdot \sec^2\theta\, d\theta \qquad ....(i)$$

Use integration by parts for $\int 2\theta \sec^2\theta\, d\theta$. Taking $u = 2\theta$ and $v = \sec^2\theta$.

$$\int 2\theta \sec^2\theta\, d\theta = 2\theta \int \sec^2\theta\, d\theta - \int\left(\frac{d}{d\theta}(2\theta)\int \sec^2\theta\, d\theta\right) d\theta$$

$$= 2\theta \tan\theta - \int 2\tan\theta\, d\theta$$

$$= 2\theta \tan\theta - 2\log|\sec\theta| + c$$

Substitute this value in (1), we get

$$\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx = \left[2\theta \tan\theta - 2\log|\sec\theta|\right]_0^{\frac{\pi}{4}}$$

$$= \left(2\cdot\frac{\pi}{4}\tan\frac{\pi}{4} - 2\log\left|\sec\frac{\pi}{4}\right|\right) - (0 - 2\log|\sec 0|)$$

$$= \frac{\pi}{2} - 2\log\sqrt{2} \quad (\because \sec 0 = 1 \ \log 1 = 0)$$

$$= \frac{\pi}{2} - \log 2$$

45. By using the property of definite integral evaluate the integral in : $\int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x\, dx}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}$

➥ $$I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x\, dx}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} \qquad ....(i)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right) dx}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right) + \cos^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)} \left\{\because \int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx\right.$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos^{\frac{3}{2}} x\, dx}{\cos^{\frac{3}{2}} x + \sin^{\frac{3}{2}} x} \qquad ....(ii)$$

Adding equation (i) and (ii), we get,

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x \; dx}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} + \int_0^{\frac{\pi}{2}} \frac{\cos^{\frac{3}{2}} x \; dx}{\cos^{\frac{3}{2}} x + \sin^{\frac{3}{2}} x}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\left(\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x\right)}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}}} dx$$

$$= \int_0^{\frac{\pi}{2}} dx$$

$$= [x]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2}$$

$$\therefore I = \frac{\pi}{4}$$

46. By using the property of definite integral evaluate the integral in : $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

➥ $$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx \qquad \text{....(i)}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin\left(\frac{\pi}{2} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}} dx \left\{\because \int_0^a f(x) \; dx = \int_0^a f(a - x) \; dx\right.$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx \qquad \text{....(ii)}$$

Adding equation (i) and (ii), we get,

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx + \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\cos x} + \sqrt{\sin x}}\right) dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}}\right) dx$$

π

$$= \int_0^{2} dx$$

$$= [x]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2}$$

$$\therefore \ I = \frac{\pi}{4}$$

47. By using the property of definite integral evaluate the integral in : $\int_0^{\pi} \log(1 + \cos x)\ dx$

➡ $I = \int_0^{\pi} \log\ (1 + \cos x)\ dx \qquad ....(i)$

$$I = \int_0^{\pi} \log\ (1 + \cos(\pi - x)\ dx$$

$$I = \int_0^{\pi} \log\ (1 - \cos x)\ dx \qquad ....(ii)$$

Adding equations (i) and (ii), we get,

$$2I = \int_0^{\pi} \log\ (1 + \cos x)\ dx + \int_0^{\pi} \log(1 - \cos x)\ dx$$

$$= \int_0^{\pi} \left(\log\ (1 + \cos x) + \log(1 - \cos x)\right)\ dx$$

$$= \int_0^{\pi} \log\ (1 + \cos x)\ (1 - \cos x) dx$$

$$= \int_0^{\pi} \log\ (1 - \cos^2 x)\ dx$$

$$= \int_0^{\pi} \log\ \sin^2 x\ dx$$

$$= \int_0^{\pi} 2 \cdot \log\ \sin x\ dx$$

$$= 2 \int_0^{\pi} \log\ \sin x\ dx$$

$2I = 2I_1$

$\therefore \ I = I_1$

where $I_1 = \int_0^{\pi} \log\ \sin x\ dx$

Now $\int_0^{2a} f(x)\ dx = 2 \int_0^{a} f(x)\ dx,\ f(2a - x) = f(x)$

Here $\sin\ (\pi - x) = \sin x$

$\frac{\pi}{\ }$

$$\therefore I_1 = 2\int_0^{2} \log \sin x \, dx \qquad \text{....(iii)}$$

$$I_1 = 2\int_0^{\frac{\pi}{2}} \log \sin\left(\frac{\pi}{2} - x\right) dx \quad \left\{\because \int_0^a f(x)dx = \int_0^a f(a-x)\,dx\right.$$

$$= 2\int_0^{\frac{\pi}{2}} \log \cos x \, dx \qquad \text{....(iv)}$$

Adding equations (iii) and (iv), we get,

$$2I_1 = 2\int_0^{\frac{\pi}{2}} \log \sin x \, dx + 2\int_0^{\frac{\pi}{2}} \log \cos x \, dx$$

$$= 2\int_0^{\frac{\pi}{2}} (\log \sin x + \log \cos x)\, dx$$

$$= 2\int_0^{\frac{\pi}{2}} \log (\sin x \cos x)\, dx$$

$$= 2\int_0^{\frac{\pi}{2}} \log \left(\frac{2\sin x \cos x}{2}\right) dx$$

$$\therefore I_1 = \int_0^{\frac{\pi}{2}} (\log \sin 2x - \log 2)\, dx$$

$$\therefore I_1 = \int_0^{\frac{\pi}{2}} \log \sin 2x \, dx - \int_0^{\frac{\pi}{2}} \log 2 \, dx$$

$$= I_2 - \frac{\pi}{2}\log 2 \qquad \text{....(v)}$$

where $I_2 = \int_0^{\frac{\pi}{2}} \log \sin 2x \, dx$

Put, $2x = t \Rightarrow 2dx = dt$

$x = 0$ then $t = 0$ and $x = \frac{\pi}{2}$ then $t = \pi$

$$I_2 = \int_0^{\pi} \log \sin t \, \frac{dt}{2}$$

$$= \frac{1}{2}\int_0^{\pi} \log \sin t \, dt$$

$$= \frac{1}{2}\int_0^{\pi} \log \sin x \, dt$$

(Replace variable t as x)
($\because$ Defenite integral does not depend on variable)

$$= \frac{1}{2} \times 2 \int_0^{\frac{\pi}{2}} \log \sin x \, dx \left[\therefore \int_0^a f(x)\,dx = 2 \int_0^{\frac{a}{2}} f(x)dx,\ f(a-x) = f(x) \right]$$

$$= \int_0^{\frac{\pi}{2}} \log \sin x \, dx$$

$\therefore I_2 = \frac{I_1}{2}$ (From (iii)) Substitute the value of I_2 in (V), we get

$$I_1 = \frac{I_1}{2} - \frac{\pi}{2} \log 2$$

$$\therefore I_1 - \frac{I_1}{2} = -\frac{\pi}{2} \log 2 \Rightarrow I_1 = -\pi \log 2$$

But $I = I_1$

$$\therefore I = -\pi \log 2$$