CHAPTER 1 & 2

OPEN STUDENT FOUNDATION
Std 12 : MATHS
PRACTICE SHEET DAY 1

Date : 18/02/24

Section A

- **Choose correct answer from the given options. [Each carries 1 Mark]** **[5]**

1. Let A = {1, 2, 3}. Then number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is
   (A) 1 (B) 2 (C) 3 (D) 4

2. Let A = {1, 2, 3}. Then number of equivalence relations containing (1, 2) is
   (A) 1 (B) 2 (C) 3 (D) 4

3. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is equal to
   (A) $\frac{7\pi}{6}$ (B) $\frac{5\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

4. $\sin\left(\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right)$ is equal to
   (A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) 1

5. $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$, then x is equal to
   (A) $0, \frac{1}{2}$ (B) $1, \frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$

Section B

- **Write the answer of the following questions. [Each carries 1 Mark]** **[5]**

1. Show that the function $f : R \rightarrow R$ given by $f(x) = x^3$ is injective.

2. Prove that : $\tan^{-1}\frac{63}{16}=\sin^{-1}\frac{5}{13}+\cos^{-1}\frac{3}{5}$.

3. Prove that : $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\frac{x}{2}, x\in\left(0,\frac{\pi}{4}\right)$

4. Prove that : $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x,\ -\frac{1}{\sqrt{2}}\leq x\leq 1$. [Hint: Put $x=\cos 2\theta$]

5. Solve the following equation : $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x,\ x\in[0,1]$

Section C

- **Write the answer of the following questions. [Each carries 2 Marks]** **[6]**

6. Find the value of the following : $\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$

7. Write the following function in the simplest form : $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right),\ 0<x<\pi$

8. Find the principal value of the following : $3\cos^{-1}x=\cos^{-1}(4x^3-3x),\ x\in\left[\frac{1}{2},1\right]$

Section D

- **Write the answer of the following questions. [Each carries 3 Marks]** **[15]**

9. Find the number of all onto functions from the set $\{1, 2, 3, \ldots, n\}$ to itself.

10. Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f : A \to B$ defined by, $f(x) = \left(\frac{x-2}{x-3}\right)$ Is f one-one and onto ? Justify your answer.

11. Let $f : N \to N$ be defined by

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}, \text{ for all } n \in N.$$

State whether the function f is bijective. Justify your answer.

12. Write the following function in the simplest form : $\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right)$, $a > 0$; $\frac{-a}{\sqrt{3}} < x < \frac{a}{\sqrt{3}}$

13. Find the value of the following : $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right]$, $|x| < 1$, $y > 0$ and $xy < 1$.

Section E

- **Write the answer of the following questions. [Each carries 4 Marks]** **[4]**

14. In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer.

(i) $f : R \to R$ defined by $f(x) = 3 - 4x$

(ii) $f : R \to R$ defined by $f(x) = 1 + x^2$

Section A

- **Choose correct answer from the given options. [Each carries 1 Mark]** **[5]**

1. Let A = {1, 2, 3}. Then number of relations containing (1, 2) and (1, 3) which are reflexive and symmetric but not transitive is

(A) 1 (B) 2 (C) 3 (D) 4

Ans. (A) 1

➥ A = {1, 2, 3}

Relation R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (1, 3), (3, 1)}

R is reflexive as (1, 1), (2, 2), (3, 3) $\in$ R

$$\left.\begin{array}{l}(1, 2) \in R \Rightarrow (2, 1) \in R \\ (1, 3) \in R \Rightarrow (3, 1) \in R\end{array}\right\} \Rightarrow R \text{ is symmetric.}$$

But (2, 1) $\in$ R, (1, 3) $\in$ A $\Rightarrow$ (2, 3) $\notin$ R

$\therefore$ R is not transitive.

$\therefore$ The number of required relation is 1.

$\therefore$ (A) will be come for answer.

2. Let A = {1, 2, 3}. Then number of equivalence relations containing (1, 2) is

(A) 1 (B) 2 (C) 3 (D) 4

Ans. (B) 2

➥ A = {1, 2, 3}

R_1 ={(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (1, 3), (3, 1), (2, 3), (3, 2)}

(1, 1), (2, 2), (3, 3) $\in R_1 \Rightarrow R_1$ is reflexive.

$$\left.\begin{array}{l}(1, 2) \in R_1 \Rightarrow (2, 1) \in R_1 \\ (1, 3) \in R_1 \Rightarrow (3, 1) \in R_1 \\ (2, 3) \in R_1 \Rightarrow (3, 2) \in R_1\end{array}\right\} \Rightarrow R_1 \text{ is symmetric.}$$

$(1, 2)(2, 1) \in R_1 \Rightarrow (1, 1) \in R_1$
$(1, 2)(2, 3) \in R_1 \Rightarrow (1, 3) \in R_1$
$(2, 3)(3, 2) \in R_1 \Rightarrow (2, 2) \in R_1$

$\therefore$ R_1 is transitive.

Thus, R_1 is reflexive, symmetric and transitive. So it is equivalence relation.

Now R_2 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)}

It is clear that R_2 is also reflexive, symmetric and transitive. So it is also equivalence relation.

Hence, the number of equivalence relations containing (1, 2) is 2.

$\therefore$ (B) will be come for the answer.

3. $\cos^{-1}\left(\cos \frac{7\pi}{6}\right)$ is equal to

(A) $\frac{7\pi}{6}$ (B) $\frac{5\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

Ans. (B) $\frac{5\pi}{6}$

➥ $\cos^{-1}(\cos x) = x,\ x \in [0, \pi]$

$$\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = \cos^{-1}\left(\cos\left(\pi + \frac{\pi}{6}\right)\right)$$

$$= \cos^{-1}\left(-\cos\frac{\pi}{6}\right)$$

$$= \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$= \pi - \cos^{-1}\left(+\frac{\sqrt{3}}{2}\right)$$

$$= \pi - \frac{\pi}{6} \quad \left\{\cos^{-1}(-x) = \pi - \cos^{-1}x\right\}$$

$$= \frac{5\pi}{6}$$

4. $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) 1

Ans. (D) 1

➥ $\sin\left(\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right)$

$$= \sin\left(\frac{\pi}{3} - \left(-\frac{\pi}{6}\right)\right)$$

$$= \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right) = \sin\left(\frac{3\pi}{6}\right) = \sin\left(\frac{\pi}{2}\right) = 1$$

5. $\sin^{-1}(1 - x) - 2\sin^{-1}x = \frac{\pi}{2}$, then x is equal to

(A) $0, \frac{1}{2}$ (B) $1, \frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$

Ans. (A) $0, \frac{1}{2}$

➥ $\sin^{-1}(1 - x) - 2\sin^{-1}x = \frac{\pi}{2}$

Let $\sin^{-1}x = \theta$

$\therefore\ x = \sin\theta$

$\therefore\ \sin^{-1}(1 - x) - 2\theta = \frac{\pi}{2}$

$\therefore\ \sin^{-1}(1 - x) = \frac{\pi}{2} + 2\theta$

$$\therefore \quad 1 - x = \sin\left(\frac{\pi}{2} + 2\theta\right)$$

$$= \cos(2\theta)$$

$$= 1 - 2\sin^2\theta$$

$$\therefore \quad 1 - x = 1 - 2x^2$$

$$\therefore \quad 2x^2 - x = 0$$

$$\therefore \quad x(2x - 1) = 0$$

$$\therefore \quad x = 0, \; x = \frac{1}{2}$$

Section B

- **Write the answer of the following questions. [Each carries 1 Mark]** **[5]**

1. Show that the function $f : \mathrm{R} \to \mathrm{R}$ given by $f(x) = x^3$ is injective.

➥ $f : \mathrm{R} \to \mathrm{R}, \; f(x) = x^3$

$x_1, x_2 \in \mathrm{R}, \; f(x_1) = f(x_2)$

$\Rightarrow x_1^3 = x_2^3$

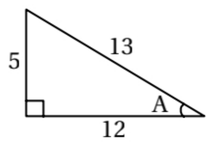
$\Rightarrow x_1 = x_2$

Hence, f is one one function.

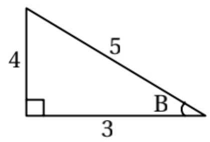
2. Prove that : $\tan^{-1}\frac{63}{16} = \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}$.

➥ Let $\sin^{-1}\frac{5}{13} = \mathrm{A}$ and $\cos^{-1}\frac{3}{5} = \mathrm{B}$

$\therefore \sin \mathrm{A} = \frac{5}{13}$ and $\cos \mathrm{B} = \frac{3}{5}$



$\therefore \tan \mathrm{A} = \frac{5}{12}$



$\tan \mathrm{B} = \frac{4}{3}$

$$\text{Now } \tan(\mathrm{A} + \mathrm{B}) = \frac{\tan \mathrm{A} + \tan \mathrm{B}}{1 - \tan \mathrm{A} \cdot \tan \mathrm{B}}$$

$$= \frac{\frac{5}{12} + \frac{4}{3}}{1 - \left(\frac{5}{12}\right)\left(\frac{4}{3}\right)}$$

$$= \frac{15 + 48}{36 - 20}$$

$$= \frac{63}{16}$$

$$\therefore \mathrm{A} + \mathrm{B} = \tan^{-1}\frac{63}{16}$$

$$\therefore \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5} = \tan^{-1}\frac{63}{16}$$

3. Prove that : $\cot^{-1}\left(\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)=\dfrac{x}{2}, x\in\left(0,\dfrac{\pi}{4}\right)$

➥ L.H.S. $= \cot^{-1}\left(\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$

$$= \cot^{-1}\left(\frac{\sqrt{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}+\sqrt{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}}{\sqrt{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}-\sqrt{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}}\right)$$

$$= \cot^{-1}\left(\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)+\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)-\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}\right) \quad \left\{\begin{array}{c}\because\ 0<x<\frac{\pi}{4}\\ 0<\frac{x}{2}<\frac{\pi}{8}\\ \therefore\ \cos\frac{x}{2}>\sin\frac{x}{2}\end{array}\right\}$$

$$= \cot^{-1}\left(\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right)$$

$$= \cot^{-1}\left(\cot\frac{x}{2}\right)$$

$$= \frac{x}{2}$$

= R.H.S.

4. Prove that : $\tan^{-1}\left(\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\dfrac{\pi}{4}-\dfrac{1}{2}\cos^{-1}x,\ -\dfrac{1}{\sqrt{2}}\le x\le 1.$ [Hint: Put $x = \cos 2\theta$]

➥ Let $x = \cos 2\theta;\ -\dfrac{1}{2} \le x \le 1$ ...(1)

$$-\frac{1}{\sqrt{2}} \le x \le 1$$

$$\Rightarrow -\frac{1}{\sqrt{2}} \le \cos 2\theta \le 1$$

$$\Rightarrow \cos\left(\frac{3\pi}{4}\right) \le \cos 2\theta \le \cos\theta$$

$$\Rightarrow \frac{3\pi}{4} \ge 2\theta \ge 0 \qquad (\cos \text{ is } \downarrow)$$

$$\Rightarrow 2\theta \in \left(0, \frac{3\pi}{4}\right)$$

$$\Rightarrow \theta \in \left(0, \frac{3\pi}{8}\right) \qquad ...(2)$$

$$\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}} = \frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}$$

$$= \frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}$$

$$= \frac{|\cos\theta|-|\sin\theta|}{|\cos\theta|+|\sin\theta|} \quad \left(\because\ \sqrt{x^2}=|x|\right)$$

$$= \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} \qquad \text{(from 2)}$$

$$= \frac{1 - \tan\theta}{1 + \tan\theta} \qquad \left(\because\ \theta \neq \frac{\pi}{2}\right)$$

$$= \tan\left(\frac{\pi}{4} - \theta\right)$$

Now, $0 < \theta < \dfrac{3\pi}{8}$

$$\Rightarrow\ \theta > -\theta - \frac{3\pi}{8}$$

$$\Rightarrow\ \frac{\pi}{4} > \frac{\pi}{4} - \theta > \frac{\pi}{4} - \frac{3\pi}{8}$$

$$\Rightarrow\ \frac{\pi}{4} > \frac{\pi}{4} - \theta > -\frac{\pi}{8}$$

$$\Rightarrow\ -\frac{\pi}{8} < \frac{\pi}{4} - \theta < \frac{\pi}{4}$$

$$\Rightarrow\ \frac{\pi}{4} - \theta \in \left(-\frac{\pi}{8}, \frac{\pi}{4}\right)$$

$$\Rightarrow\ \frac{\pi}{4} - \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \qquad \ldots(3)$$

Thus $\dfrac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}} = \tan\left(\dfrac{\pi}{4} - \theta\right);\ \ \dfrac{\pi}{4} - \theta \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

$$\tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \theta\right)\right)$$

$$= \frac{\pi}{4} - \theta \qquad \text{(from 3)}$$

$$= \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x \qquad \text{(from 1, 2)}$$

5. Solve the following equation : $\tan^{-1}\left(\dfrac{1-x}{1+x}\right) = \dfrac{1}{2}\tan^{-1} x,\ \ x \in [0, 1]$

➥ $\tan^{-1}\left(\dfrac{1-x}{1+x}\right) = \dfrac{1}{2}\tan^{-1} x$

$$\therefore\ \tan^{-1}1 - \tan^{-1}x = \frac{1}{2}\tan^{-1} x$$

$$\therefore\ \frac{\pi}{4} = \frac{1}{2}\tan^{-1} x + \tan^{-1} x$$

$$\therefore\ \frac{\pi}{4} = \frac{3}{2}\tan^{-1} x$$

$$\therefore\ \tan^{-1} x = \frac{\pi}{4} \times \frac{2}{3} = \frac{\pi}{6}$$

$$\therefore\ x = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

Section C

- **Write the answer of the following questions. [Each carries 2 Marks]** **[6]**

6. Find the value of the following : $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

➥ We know that,

Range of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Range of $\cos^{-1}$ is $[0, \pi]$ and

Range of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$\therefore \tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

$= \frac{\pi}{4} + \left(\pi - \frac{\pi}{3}\right) + \left(-\frac{\pi}{6}\right)$

$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$

$= \frac{3\pi + 8\pi - 2\pi}{12}$

$= \frac{9\pi}{12} = \frac{3\pi}{4}$

7. Write the following function in the simplest form : $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$, $0 < x < \pi$

➥ $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right) = \tan^{-1}\left(\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}\right)$

$= \tan^{-1}\left(\sqrt{\tan^2\frac{x}{2}}\right)$

$= \tan^{-1}\left(\tan\frac{x}{2}\right)$

Here, $0 < x < \pi \Rightarrow$ we have $0 < \frac{x}{2} < \frac{\pi}{2}$

$= \frac{x}{2}$

8. Find the principal value of the following : $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$, $x \in \left[\frac{1}{2}, 1\right]$

➥ Let $\cos^{-1}x = \theta;\ \frac{1}{2} \le x \le 1$ ...(1)

$\Rightarrow \cos^{-1}\left(\frac{1}{2}\right) \ge \cos^{-1}(x) \ge \cos^{-1}(1)$ $\{\cos^{-1} \downarrow$ function$\}$

$\Rightarrow \frac{\pi}{3} \ge \theta \ge 0$

$\Rightarrow 0 \le \theta \le \frac{\pi}{3}$ ...(2)

$\Rightarrow\ 0 \le 3\theta < \pi$ ...(3)

Now, $\cos^{-1}x = \theta,\ \frac{1}{2} \le x \le 1,\ 0 \le \theta \le \frac{\pi}{3}$

$\Rightarrow\ x = \cos\theta$...(4)

R.H.S. $= \cos^{-1}(4x^3 - 3x)$

$= \cos^{-1}(4\cos^3\theta - 3\cos\theta)$

$= \cos^{-1}(\cos 3\theta)$

$= 3\theta$ (from (3))

$= 3\cos^{-1}x$ (from (1))

= L.H.S.

Section D

- **Write the answer of the following questions. [Each carries 3 Marks]** **[15]**

9. Find the number of all onto functions from the set $\{1, 2, 3, \ldots, n\}$ to itself.

➥ The set A = $\{1, 2, 3, \ldots\ldots, n\}$ = A

The set A has n element.

$\therefore$ The number of all onto function from A to A is $n(n-1)(n-2)\ldots\ldots 1 = n!$

10. Let A = R − {3} and B = R − {1}. Consider the function f : A → B defined by, $f(x) = \left(\frac{x-2}{x-3}\right)$ Is f one-one and onto ? Justify your answer.

➥ A = R − {3} and B = R − {1}

$f : A \to B,\ f(x) = \frac{x-2}{x-3}$

$x_1, x_2 \in A\ \ f(x_1) = f(x_2)$

$\therefore \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$

$\therefore (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

$\therefore x_1x_2 - 2x_2 - 3x_1 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

$\therefore 3x_2 - 2x_2 = 3x_1 - 2x_1$

$\therefore x_2 = x_1$

$\therefore x_1 = x_2$

$\therefore$ The function f is one one.

Let $y \in$ R − {1}

$y = f(x) = \frac{x-2}{x-3}$

$\therefore y(x-3) = x - 2$

$\therefore xy - 3y = x - 2$

$\therefore x(y-1) = 3y - 2$

$\therefore x = \dfrac{3y-2}{y-1},\ y \neq 1$

Now $f(x) = f\left(\dfrac{3y-2}{y-1}\right) = \dfrac{\frac{3y-2}{y-1} - 2}{\frac{3y-2}{y-1} - 3}$

$$= \frac{3y - 2 - 2y + 2}{3y - 2 - 3y + 3}$$

$$= y$$

$\therefore$ The function f is onto.

11. Let $f : \text{N} \to \text{N}$ be defined by

$$f(n) = \begin{cases} \dfrac{n+1}{2}, & \text{if } n \text{ is odd} \\ \dfrac{n}{2}, & \text{if } n \text{ is even} \end{cases},\ \text{for all } n \in \text{N}.$$

State whether the function f is bijective. Justify your answer.

➥

$$f : \text{N} \to \text{N},\ f(n) = \begin{cases} \dfrac{n+1}{2}, & \text{if } n \text{ is odd.} \\ \dfrac{n}{2}, & \text{if } n \text{ is even.} \end{cases},\ \text{for all } n \in \text{N}.$$

$f(3) = \dfrac{3+1}{2} = 2$ ($\because$ 3 is odd)

$f(4) = \dfrac{4}{2} = 2$ ($\because$ 4 is even)

That is $f(2m-1) = \dfrac{2m-1+1}{2} = m$

and $f(2m) = \dfrac{2m}{2} = m,\ m \in \text{N}$.

$\therefore$ f is not one one function.

Range of f = N.

Each element of co-domain of f is on image of any element of domain.

$\therefore$ f is onto.

Thus, f is onto function but it is not one one function.

$\therefore$ f is not bijective.

12. Write the following function in the simplest form : $\tan^{-1}\left(\dfrac{3a^2x - x^3}{a^3 - 3ax^2}\right),\ a > 0;\ \dfrac{-a}{\sqrt{3}} < x < \dfrac{a}{\sqrt{3}}$

➥ Let $x = a \tan\theta$

$\Rightarrow \tan\theta = \dfrac{x}{a}$ $(a > 0)$

$\Rightarrow \theta = \tan^{-1}\dfrac{x}{a}$

$\tan^{-1}\left(\dfrac{3a^2x - x^3}{a^3 - 3ax^2}\right)$

$= \tan^{-1}\left(\dfrac{3a^2 \cdot a\tan\theta - a^3\tan^3\theta}{a^3 - 3a \cdot a^2\tan^2\theta}\right)$

$= \tan^{-1}\left(\dfrac{3a^3\tan\theta - a^3\tan^3\theta}{a^3 - 3a^3\tan^2\theta}\right)$

$= \tan^{-1}\left(\dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right)$ $\left\} \begin{array}{l} -\dfrac{a}{\sqrt{3}} < x < \dfrac{a}{\sqrt{3}}, a > 0 \\ \Rightarrow -\dfrac{1}{\sqrt{3}} < \dfrac{x}{a} < \dfrac{1}{\sqrt{3}} \end{array}\right.$

$\left.\begin{array}{l} = \tan^{-1}(\tan 3\theta) \\ = 3\theta \\ = 3\tan^{-1}\dfrac{x}{a} \end{array}\right\} \begin{array}{l} \Rightarrow \tan^{-1}\left(-\dfrac{1}{\sqrt{3}}\right) < \tan^{-1}\dfrac{x}{a} < \tan^{-1}\dfrac{1}{\sqrt{3}} \\ \Rightarrow -\dfrac{\pi}{6} < \tan^{-1}\dfrac{x}{a} < \dfrac{\pi}{6} \end{array}$

13. Find the value of the following : $\tan\dfrac{1}{2}\left[\sin^{-1}\dfrac{2x}{1+x^2} + \cos^{-1}\dfrac{1-y^2}{1+y^2}\right]$, $|x| < 1$, $y > 0$ and $xy < 1$.

➥ $\tan\dfrac{1}{2}\left[\sin^{-1}\dfrac{2x}{1+x^2} + \cos^{-1}\dfrac{1-y^2}{1+y^2}\right]$

$= \tan\dfrac{1}{2}\left[\sin^{-1}\left(\dfrac{2\tan\theta}{1+\tan^2\theta}\right) + \cos^{-1}\left(\dfrac{1-\tan^2\phi}{1+\tan^2\phi}\right)\right]$ $\left(\begin{array}{l} \because \text{Taking } x = \tan\theta \text{ and } y = \tan\phi \\ \theta = \tan^{-1}x \text{ and } \phi = \tan^{-1}y \end{array}\right)$

$= \tan\dfrac{1}{2}\left[\sin^{-1}(\sin 2\theta) + \cos^{-1}(\cos 2\phi)\right]$

$= \tan\dfrac{1}{2}[2\theta + 2\phi]$

$\left.\begin{array}{l} = \tan[\theta + \phi] \\ = \tan[\tan^{-1}x + \tan^{-1}y] \\ = \tan\left[\tan^{-1}\dfrac{x+y}{1-xy}\right] \end{array}\right\} |x| < 1, \text{y} > 0$

$= \dfrac{x+y}{1-xy}$ $(\because xy < 1)$

Section E

- **Write the answer of the following questions. [Each carries 4 Marks]** **[4]**

14. In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer.

(i) $f : R \to R$ defined by $f(x) = 3 - 4x$

(ii) $f : R \to R$ defined by $f(x) = 1 + x^2$

➥ **(i) $f : R \to R$ defined by $f(x) = 3 - 4x$**

$f : R \to R,\ f(x) = 3 - 4x$

$x_1, x_2 \in R,\ f(x_1) = f(x_2)$

$\Rightarrow 3 - 4x_1 = 3 - 4x_2$

$\Rightarrow x_1 = x_2$

$\therefore$ The function f is one one.

Let $y \in R$ (co-domain)

$\therefore\ y = f(x) = 3 - 4x$

$\therefore\ 4x = 3 - y$

$\therefore\ x = \dfrac{3 - y}{4}$

$f(x) = f\left(\dfrac{3 - y}{4}\right) = 3 - 4\left(\dfrac{3 - y}{4}\right) = y$

Thus, $\forall\, y \in$ R there exists $x \in$ R such that $f(x) = y$.

$\therefore\ f$ is onto.

Thus, the function f is one one and onto function, So, it is bijective function.

➥ **(ii) f : R → R defined by $f(x) = 1 + x^2$**

f : R → R, $f(x) = 1 + x^2$

For $x_1, x_2 \in$ R, let $f(x_1) = f(x_2)$

$\Rightarrow\ 1 + x_1^2 = 1 + x_2^2$

$\Rightarrow\ x_1^2 = x_2^2$

$\Rightarrow\ x_1 = \pm x_2$

$f(2) = 1 + (2)^2 = 5$ and $f(-2) = 1 + (-2)^2 = 5$

$\therefore\ 2 \neq -2$ but $f(2) = f(-2)$

$\therefore$ The function f is not one one.

It is clear that $1 + x^2 \geq 1$

$\therefore$ Range of f is positive real number.

If y is negative then we can not find x such that $f(x) = y$.

$\therefore\ f$ is not onto.

Thus, the function f is neither one one nor onto.

$\therefore\ f$ is not bijective function.